

EXIT Charts

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Abstract – With wide applications of Turbo Principle in communications engineering, the EXIT Charts (Extrinsic Information Transfer Charts) has been a powerful tool to visualize the convergence behavior of iterative decoding process based on mutual information. Within this report, an introduction to EXIT Charts, its background, properties and applications are presented to allow the audience be familiar with this useful and insightful analysis tool when iterative decoding deployed.

1. INTRODUCTION

The discovery of Turbo codes [1] in 1993 by a French research group, has ignited the information society and has been leading to many years of academic research and discussion in Error Control Coding realm, due to the powerful performance of Turbo codes in terms of coding gain. It presents a skillful technique to approach the Shannon limit very closely under the intuitive insight of feedback loop. Later on, the Turbo Principle [2] is proposed whilst widely accepted and utilized to a variety of applications based on its underlying concept. Furthermore, the original parallel concatenated structure has been extended to serial concatenated structure, which is commonly referred in the Turbo Principle. A powerful and skillful analysis tool, EXIT Charts (Extrinsic Information Transfer Charts), for the purpose of analyzing the iterative decoding convergence in Turbo Codes and Turbo related codes [3] is originally created by Stephen ten Brink [4]. This report presents an introduction to this powerful and insightful tool of EXIT Charts, its properties and applications as well.

2. THE INVENTION OF EXIT CHARTS

2.1 Introduction and motivation of EXIT Charts

Although the Turbo codes enable to approach the Shannon limit very closely, there does not exist a satisfactory analysis tool designed to analyze the convergence behavior, particularly when iterative decoding process is deployed. The conventional analysis tool BER (Bit Error Rate) Charts cannot completely reflect the improvement of iterative decoding process when the number of iterations increases. As showed in Fig. 1, the performance of BER Charts can be divided into three regions. In the high signal to noise ratio (SNR) region BER Charts performs well, while in the low SNR region and Turbo cliff region BER Charts could not reflect how much can be improved when the number of iterations increases. This basic phenomenon becomes the original motivation for EXIT Charts invention. In Section 4 the comparison between EXIT Charts and BER Charts will be demonstrated.

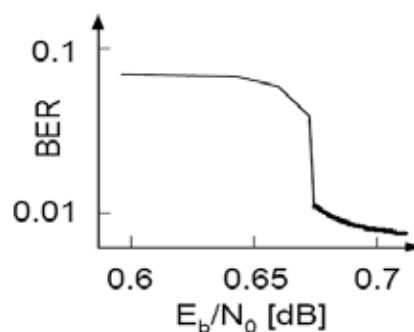


Fig.1 BER Charts when deploying iterative decoding process

2.2 Turbo Principle

Turbo principle comes from Turbo codes, because the iterative feedback concept involved with Turbo codes can be applied to a variety of detection and decoding problems, such as serial concatenation, equalization, coded modulation, joint source-channel decoding, multiuser detection and others. If there

exist two or more decoders, Turbo Principle can perform maximum-a-posteriori (MAP) estimations of symbols, because the priori knowledge to each decoder will be updated after every iteration.

2.2.1 Log-Likelihood Algebra

Log-likelihood values are commonly used when taking Turbo Principle into account, due to the simplification of calculation and succinct expressions of output value brought with L-values. Let U be in GF(2) with elements $\{+1, -1\}$, the log-likelihood ratio of a binary random variable U , $L_U(u)$, is defined as

$$L_U(u) = \log \frac{P_U(u=+1)}{P_U(u=-1)}.$$

Here the P_U denotes the probability of random variable U when taking the value of u . The log-likelihood ratio $L_U(u)$ presents the L-value of random variable U . The sign of $L_U(u)$ is the hard decision, while the magnitude $|L_U(u)|$ shows the reliability of this value. If the binary random variable is conditioned on another random variable or vector y , then conditioned log-likelihood ratio is $L(u|y) = L(u) + L(y|u)$, when Bayes rule is utilized. When we use

$$P(u = \pm 1) = \frac{e^{\pm L(u)}}{1 + e^{\pm L(u)}}$$

for statistical independent random variable U_1, U_2 , we can get

$$\begin{aligned} L(u_1 \oplus u_2) &= \log \frac{1 + e^{L(u_1)} e^{L(u_2)}}{e^{L(u_1)} + e^{L(u_2)}} \\ &\approx \text{sign}(L(u_1)) \cdot \text{sign}(L(u_2)) \cdot \min(|L(u_1)|, |L(u_2)|). \end{aligned}$$

2.2.2 Turbo Principle based on L-values

With the L-values, we can perform Turbo Principle in a much simpler formula. After transmission over a binary symmetric channel (BSC) or a Fading Channel, the log-likelihood ratio of the transmitted bit x conditioned on the output y from matched filter can be calculated as

$$\begin{aligned} L(x|y) &= \log \frac{P(x=+1|y)}{P(x=-1|y)} \\ &= \log \frac{\exp(-\frac{E_s}{N_0}(y-a)^2)}{\exp(-\frac{E_s}{N_0}(y+a)^2)} + \log \frac{P(x=+1)}{P(x=-1)} \\ &= L_c \cdot y + L(x), \end{aligned}$$

where L_c is the channel state information and $L(x)$ is the a priori information. Furthermore, for statistical independent transmission, such as dual diversity or repetition code, we have

$$L(x|y_1, y_2) = L_{c1} \cdot y_1 + L_{c2} \cdot y_2 + L(x).$$

A turbo decoder accepts both a priori and channel L-values and delivers soft-output L-values $L(\hat{u})$.

Besides this the so-called extrinsic L-values for the information bits $L_e = (\hat{u})$ is produced. Extrinsic information refers to the additional information obtained through decoding process about all the other current bits. Only this extrinsic information will be utilized as a priori information and will be sent to the other decoder after interleaver or deinterleaver, this action avoids the propagation of information already known. Simply put, $L(\hat{u}) = L_c \cdot y + L(u) + L_e(\hat{u})$, it means that the a posteriori L-values contains three terms, which are the channel state information, the a priori information and extrinsic information. As showed in Fig.2, the MAP estimate contains three parts: from the channel, from the a priori information and from other bits through constraints of the code or Markov property.

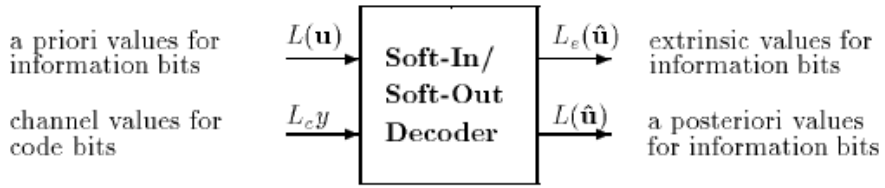


Fig.2 Soft-in/Soft-out decoder for turbo iteration

2.3 Introduction to parallel and serial concatenated iterative decoders

Turbo codes are also called as the parallel concatenated convolutional codes (PCCC). The basic iterative decoder for parallel concatenated codes is showed in Fig. 3, where i is the systematic bits, p_1, p_2 are parity check bits.

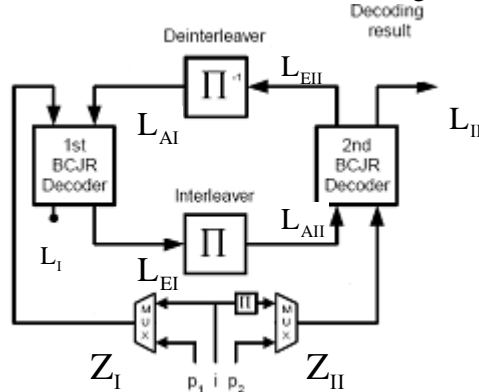


Fig. 3 Iterative decoder used in parallel concatenated codes

The channel observations Z_I will be sent to the first iterative decoder, which uses BCJR algorithm to decode. The soft-output from the first BCJR decoder deploys the same Soft-input/Soft-output function as showed in Fig. 2. The extrinsic information from the first decoder will be sent to the second decoder. This extrinsic information becomes the a priori information to the second decoder after passing through an interleaver. The second decoder implements the same function as the first decoder. Soft-output L_{II} is regarded as the decoding result, while extrinsic information L_{EII} will become the a priori information for the first decoder after its passing through a deinterleaver.

Although the turbo codes was originally proposed as parallel concatenations, the Turbo Principle can be also applied in serial concatenations as presented in Fig.4. The basic iterative decoding principle, inclusive of producing extrinsic knowledge from one decoder as a priori knowledge for the other decoder and Soft-input/Soft-output algorithm, is the same as parallel concatenations. However, one thing to be paid attention is that the observation from the channel is only once instead of twice, since the information bits only pass one individual iterative decoder. As a result, an estimate channel is required to align with the communications channel in order to implement the BCJR algorithm.

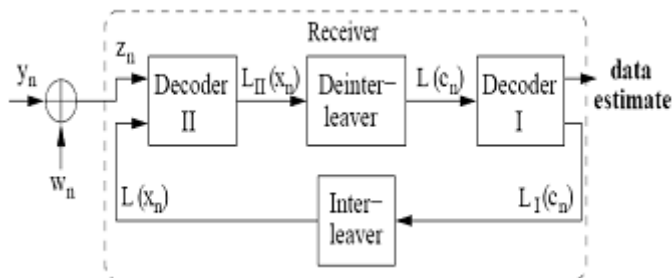


Fig. 4 Iterative decoder used in serial concatenated codes

3. THE CONSTRUCTION OF EXIT CHARTS

EXIT Chart is a powerful tool to visualize the convergence behavior of iterative decoding algorithms based on mutual information. The term of mutual information in information theory proposed by

Claude E. Shannon in 1948 [8] decides the uncertainty resolved on observing other conditional information. Channel Capacity is the maximum value of mutual information over all the possibilities of input probability distribution. The choice of mutual information as the parameter to plot EXIT Chart is based on the following reasons:

1. Robustness: the shape of transfer characteristic is quite robust against the shape of the a priori input distribution, since the first few a priori input distribution are not Gaussian distributions;
2. Theoretical interpretation: Shannon tells that the mutual information between the channel input and variable output relates to the maximum possible transmission rate for reliable transmission;
3. The value range and logarithm calculation lead to an easier way to plot the EXIT Charts.

3.1 Transfer characteristics of the iterative decoder

As mentioned in Section 2, iterative decoders exchange messages. Moreover, the tracking of messages only depends on the mutual information between L-values extrinsic information and the a priori information about the transmitted information bits. Viewing I_E as a function of I_A and the E_b / N_0 value, the extrinsic information transfer characteristics are defined as $I_E = T(I_A, E_b / N_0)$, if the E_b / N_0 is fixed, just $I_E = T(I_A)$. By using Monte Carlo simulations, the function plot between I_A and I_E can be observed. Several parameters influence the transfer characteristic curves, such as the number of code memory, the depth of interleaving, different code polynomials as well as the fixed value of E_b / N_0 . Take Fig. 5 as an example, the trend that I_E will be monotonically increased with increasing I_A can be perceived.

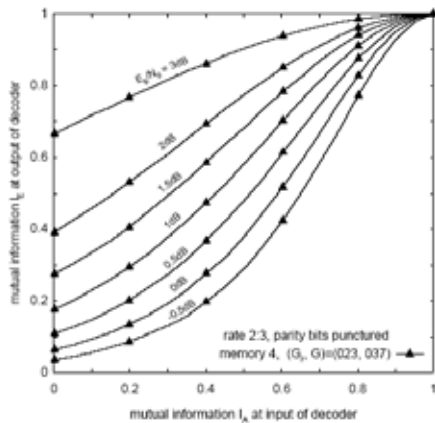


Fig. 5 Extrinsic information transfer characteristics of Soft-in/Soft-out decoder for rate 2/3 convolutional code, E_b / N_0 of channel observations as parameter to curves

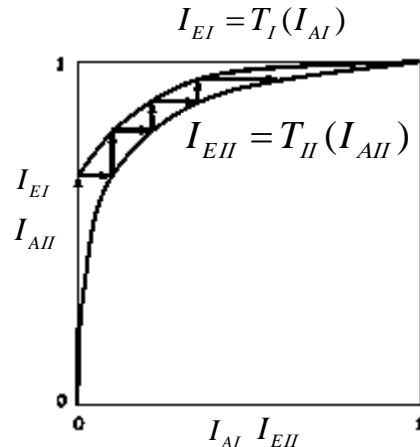


Fig. 6 Trajectory of iterative decoding

3.2 Combination of two transfer characteristics

Since two decoders are involved in iterative decoding, there exist two transfer characteristics of each decoder. Due to the Turbo Principle, one decoder output the L-values mutual information of extrinsic information as the a priori knowledge to the second decoder after passing de/interleaver, the same way to the other decoder. As a result, the combination of two transfer characteristics gives rise to the EXIT Charts. The EXIT Charts not only show transfer characteristic of each decoder, but also visualize the iterative decoding process by stair-case curves presented in EXIT Charts. Fig. 6 shows how to follow the iterative decoding trajectory when observing EXIT Charts.

The lower curve presents the transfer characteristic of the second decoder, while the upper curve shows that of the first decoder. The output extrinsic information from the first decoder becomes the a

priori information to the second decoder after passing deinterleaver, whereas the extrinsic information from the second decoder turn to the a priori information to the first decoder after passing interleaver.

3.3 Two simplified consequences from L-values with serial concatenations

When the L-values algebra is adopted, the calculation of mutual information can be greatly simplified due to two direct consequences from L-values. One consequence is that the probability density function is symmetric

$$p(-y|x=+1) = p(y|x=-1),$$

for APP (A Posterior Probability) based decoders of any linear code, this condition can always be fulfilled. The other consequence is consistency condition

$$p(-y|x) = e^{-L_{xy}} p(y|x),$$

when the LLR calculations are correct, this consequence can be met. Due to these simplifications, the calculation of mutual information becomes to

$$\begin{aligned} I(L; X) &= 1 - \int_{-\infty}^{+\infty} p(L|x=+1) \log_2(1 + e^{-L}) dL \\ &= 1 - E\{\log_2(1 + e^{-L})\}. \end{aligned}$$

Because the mutual information between L-value and transmitting bits can be expressed in a general form as showed above, we can derive a machine which outputs both the horizontal and vertical value as showed in Fig. 7.

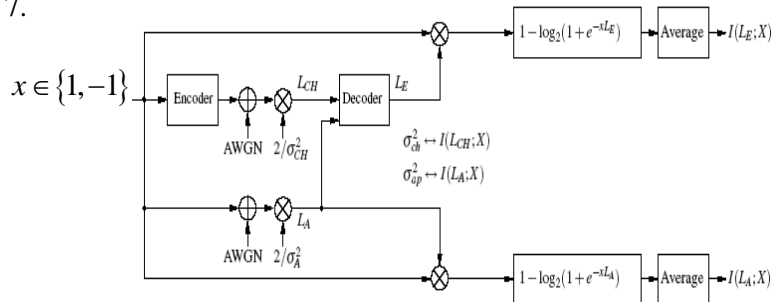


Fig 7. Measurement of EXIT Charts

4. ADVANTAGES AND FEATURES OF EXIT CHARTS

4.1 Comparison with BER Charts

As already discussed in Section 2.1, EXIT Charts can overcome some disadvantages of BER Charts when dealing with iterative decoding. EXIT Charts can reflect the three SNR regions in BER Charts. As seen in Fig.8, the trajectory is stuck at low mutual information at very low SNR region or pinch-off region. Turbo cliff region is the bottleneck region with slow convergence. BER floor region is the

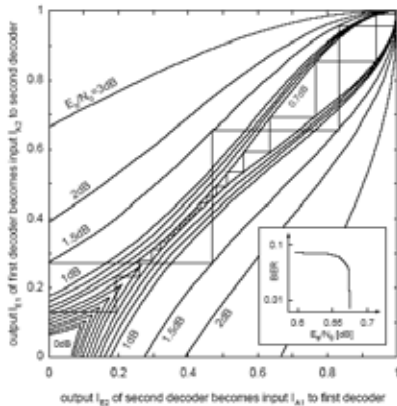
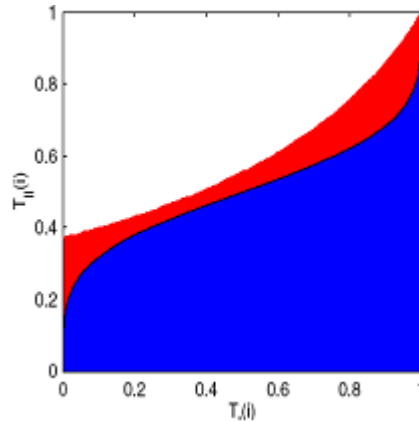


Fig. 8 Compare EXIT Charts with BER Charts



$$\begin{aligned} \int_0^1 T_{II}(i) di &\approx C_{ui} \\ \int_0^1 T_I^{-1}(i) di &\approx R_I \\ \int_0^1 (T_{II}(i) - T_I^{-1}(i)) di &> 0 \\ \Rightarrow R_I &< C_{ui} \end{aligned}$$

Fig. 9 Area Property of EXIT Charts

wide-open region with fast convergence. Meanwhile, when the two transfer characteristics curve intersect with each other, the decoding process will stop there. Therefore, the acronym EXIT can also be understood as the opportunity to see at what E_b/N_0 value, the decoding trajectory can successfully go through the bottleneck toward low BER when there exists the pinch-off region.

4.2 Properties of EXIT Charts

Besides enabling to show the decoding trajectory of iterative decoding, EXIT Charts own many other merits for further coding researches. One interesting property is Rate-Capacity properties for serial concatenated schemes. It coincides with Channel Coding theory. Code rate of outer code should be smaller than the capacity of inner channel. Fig.9 just gives a simple example to show this property.

5. APPLICATION OF EXIT CHARTS

There are a variety of applications of EXIT Charts. Here only Single Parity Check Codes is given below, used to justify the questions and issues we mentioned before. EXIT Charts can also be used in LDPC irregular codes.

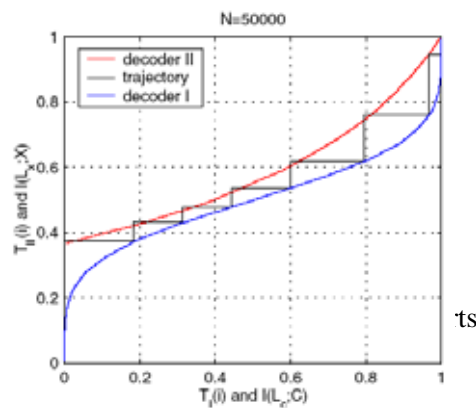


Fig. 10 Simple example of EXIT Charts

6. CONCLUSIONS

The background and motivation of EXIT Charts invention are briefly introduced. Turbo Principle and its feedback concept are presented under both parallel and serial concatenated structure. As a powerful tool to visualize the convergence behavior of iterative decoding explicitly, how to construct EXIT Charts, comparison with BER Charts, properties and one simple application are given. EXIT Charts provides us some insight on code design in the future researches. The fruitful applications and promising prospect will enlighten researchers to further dig and absorb more information about Turbo-related codes.

REFERENCES

- [1] C.Berrou, A. Glavieux and P. Thitimajshima, "Near Shannon limit error correcting coding and decoding: Turbo codes (1)," *Proc. ICC '93*, May 1993, pp. 1064-1070.
- [2] J. Hagenauer, "The Turbo Principle: Tutorial introduction and state of the art" in *Proceedings of the international Symposium on Turbo Codes*, Brest, France, September 1997, pp. 1-11.
- [3] J.Sayir, G.Lechner, "Theory and Design of Turbo and Related Codes", <http://userver.ftw.at/~jossy/>.
- [4] S. ten Brink, "Convergence of iterative decoding" in *Electronics Letters*, Vol 35, No.10, May 1999.
- [5] J. Hagenauer, "The EXIT Chart- Introduction to Extrinsic Information Transfer in Iterative Processing", *EUSIPCO 2004*.
- [6] S. ten Brink, "Iterative decoding trajectories of parallel concatenated codes", in *Proc. 3rd ITG Conference Source and Channel Coding*, Munich, Germany, January2000, pp. 75-80.
- [7] S. ten Brink, "Convergence Behavior of Iteratively Decoded Parallel Concatenated Codes", *IEEE Trans. Comm.*, Vol. 49, Oct.2001.
- [8] Claude E. Shannon, "A Mathematical Theory of Communications" *The Bell System Technical Journal*, Vol. 27, July, October, 1948, pp.379-423, 623-656.