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Investigation on Current Mills and Wind Turbines

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1 Introduction

In times of increasing costs for energy, the lack of sustainability of fossil or nuclear fuels and the will to reduce greenhouse gas emissions, the use of renewable energy resources is getting more and more important for a sustainable future. Possibilities are solar or hydro power or the use of biomass for power generation. Another important renewable energy source, that already plays an important role, is the use of wind power. In Comparison to these well-known ideas, extracting energy from tidal currents seems to be a relativly new approach. The following pages shall give a closer look about machines and technologies for energy conversion from tidal currents and wind streams.
Before we start using the energy, it is important to know how it is generated in nature. Due to the geographical differences on earth, areas of lower and higher pressure occure in the atmosphere. As these pressure differences try to compensate, wind streams are generated.
The physical reasons for tidal currents are based on the gravitation forces within the interplanetary system between Sun, Moon and Earth. The cyclic change of the sealevel follows a sinus function with a period length of 12 hours and 25 minutes.
To use these energetic streams, there are several types fo so called free stream turbines. The main characteristic of these machines is the position of the rotating shaft. It can either be horizontal or vertical. With a special view on tidal currents, the interesting concept of the vertical axis free stream turbines will be discussed in more detail. An analytical model is derived and validated.

2 Boundary Conditions

To be able to find applicable solutions for energy conversion, a look at the boundary conditions of the systems is necessary. The direct comparison between the most important physical properties (figure 4) shows that the characteristics of the stream (here shown by the Reynolds number)

	wind	tidal currents
density ρ	1,2 kg/m³	1000-1020 kg/m³
viskosity η	0,0000171 N/(m² s)	0,001 N/(m² s)
velocity v	10 m/s	2 – 3 m/s
Re	**300000**	**450000**

Figure 1: physical stream properties

of wind and tidal currents are at least in the same order of magnitude. Because of this, we can asssume that the technologies for converting tidal currents have to be similar to those of wind turbines. Today there are lots of efficient systems for using wind power, that exactly these ideas may be applicable for tidal curent turbines, too. Another common assumption made for wind streams througt free stream turbine is the incompressibility. As we can use this simplification, the complexity of the overall calculation will decrease. As apposed to those facts above, there are some important diffenrences between tidal currents and wind streams: One main point is the predictability of the velocity. In comparison to wind streams, which are more or less stochastically distributed, the velocity in tidal currents follows strikt sinus functions. In free stream, its always necessary to consider the appearance of boundary layers. In addition to the boundary layer at the groud (ground or seabed) there is a second layer in tidal streams caused by the phase border at the ocean surface. In this special case, not all wind power concepts are suitable for tidal currents. The machine should be albe to easyly use streams from two opposite directions and it should not be sensitive to sea level changes. Here a turbine with vertical axis seems to have decent advantages and could be a possible solution.

3 Theory of Energy Conversion

In this part a short introduction into the theoretical approaches for free stream turbines is to be given. The maximum effeciency reachable by such a turbine was first calculated by Betz.

3.1 Betz' Theory

A turbine power always has to be related to an overall power available. In our case, this is the total kinetic energy per unit time flowing through the turbine area (equation 1).

$$P_{wind} = \frac{dE_{wind}}{dt} = \frac{d}{dt}(\frac{1}{2}mv_{\infty}^2) \tag{1}$$

The calculation of the turbine instead will take a few more steps. At first the equation of continuity (equation 2) and a general energy balance (equation 3) have to be applied at the streamtube before, after and directly in the turbine area.

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = \rho_3 A_3 v_3 \tag{2}$$

$$\Delta E = E_1 - E_3 = \frac{1}{2}m(v_1^2 - v_3^2) \tag{3}$$

Derivating the given energy difference over the turbine area with time gives us the turbine power depending on the velocities before and after the turbine and the mass flow rate. After substituting the mass flow rate by a term only depnding on the speed in the turbine area, the power is only a funciton of different velocities. The velocity v_2 can be cancelled out using the Froude-Rankine-Theorem [1]. We finally reach an expression for the turbine power only depending on a dimensionless velocity $\frac{v_1}{v_3}$. By obtaining the maximum of this function, we know the maximum effiency of a wind turbine, which is $\approx 59\%$ at the velocity ratio of $\frac{1}{3}$.

3.2 Airfoil Theory

A stream around an airfoil always leads to a force. This force is depending on the velocity, the density of the streaming medium and the geometry of the airfoil. In aerodynamics, this

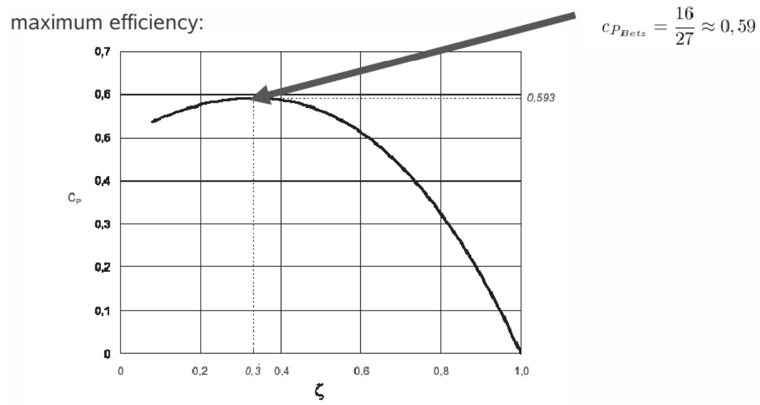

Figure 2: Betz' Efficiency

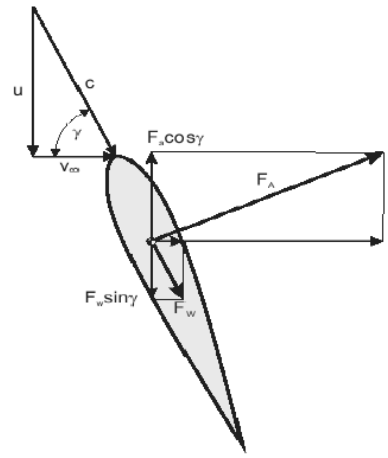

Figure 3: Velocities and forces at an airfoil

force is always divided into one force in the direction of the stream, the drag force, and one orthogonal to the stream, the lift force [2].

4 Vertical Axis Free Stream Turbine

To prove that a vertical axis turbine is a suitable solution for converting tidal currents, a comparison to horizontal axis machines has to be carried out.

4.1 Vertical vs. Horizontal Concepts

Horizontal axis turbines have the problem of big losses due to the angular momentum in low velocities [3]. This has been prooved by Schmitz. Furtheron, these kind of machines are very sensible to sea level changes. If the sealevel drops in the area of the blades, severe damage to the blades could occur and the power would decrease rapidly. The vertical axis turbine on the other hand can bear these situation, rotating in the sealevel area. Another important issue for a vertical rotating shaft is the possibility of installing the generator and the transmission above the sealevel without complex and expensive sealing or the problem of underwater maintenance. For vertical axis free stream turbines there are two different main operating modes: There are drag and lift based designs, according to the used force, appearing on the airfoil.

4.2 Drag Based Vertical Free Stream Turbines

These kind of turbines are the oldest and simplest way to use free stream energy. Good examples are the old persian windmills or an anenometer. The theory of drag based turbines is based on the examination of a disc with an Area A moving only translational with a constant velocity u in a stream with the velocity v. The power extracted by this disc is calculated according to equations 4 and 5. The force on the disc is:

$$F_W = \frac{\rho}{2} c_W A v_r el \tag{4}$$

The Power is given to:

$$P_T = F_W u = \frac{\rho}{2} c_W A(v_\infty - u)u \tag{5}$$

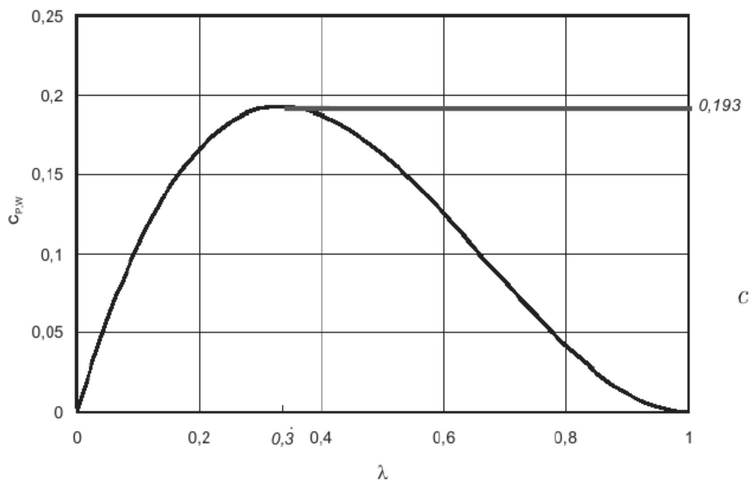

Figure 4: Maximum drag power coefficient

After making the power nondimensional again, we obtain a theoretical maximal power coefficient for drag based designs of 19,3 %. In reality this value drops down to 10% or even below due to losses.

4.3 Lift Based Vertical Free Stream Turbines

The main concept of lift based vertical axis free stream turbines was develoed by Darrieus in the 1920s, that is why these machines are often called Darrieus-turbines. The power generation is here performed by lift forces appearing at the airfoil. The force is, again, depending on the absolut stream velocity, consisting of the stream speed and the speed of the airfoil. As we calculate the power extracted by such an airfoil (equation **??**) , the efficiency is depending on the glide ratio (equation**??**).

$$P = (F_A \cos(\gamma) - F_W \sin(\gamma))u = \frac{\rho}{2} A v_\infty^3 c_A \lambda \sqrt{1+\lambda^2} \left(1 - \frac{\lambda c_W}{c_A} \right. \tag{6}$$

$$c_{P,A} = c_A \lambda \sqrt{1+\lambda^2} \left(1 - \frac{\lambda}{\frac{c_A}{c_W}} \right. \tag{7}$$

The comparison of these to designs with a plate (glide number $\epsilon = 10$) leads us to the conclusion that the efficiency of lift based turbines much higher is than of drag based designs. Now, let us focus on more realistic geometries of a vertical axis turbine as shown in figure 5. To be able to compare diffenrent types of free stream turbines, we intruduce the tip speed ratio as a dimensionsless velocity (equation 8).

$$\lambda = \frac{R\Omega}{v_1} \tag{8}$$

Now we can obtain the behaviour of the absolute velocity (equation 9) and the angle of attack (equation 10) at the airfoils depending on the azimuth angle.

$$c_a(\Theta, \lambda) = 2 \cdot \pi \cdot \sin(\alpha(\Theta, \lambda)) \tag{9}$$

$$\alpha(\Theta, \lambda) = \begin{cases} \arcsin\left(\frac{cos\Theta}{\lambda}\right) & , \frac{\pi}{2} < \Theta < \frac{3\pi}{2} \\ -\arcsin\left(\frac{cos\Theta}{\lambda}\right) & , else \end{cases} \tag{10}$$

The velocity is important for the calculation of the forces at the different blades (equation 11 and **??**). After modeling the lift and drag coefficients, forces, torques and the power of the rotor can be calculated (equation 13).

$$F_a(\Theta, \lambda) = c_a(\Theta, \lambda) \cdot \frac{\rho}{2} \cdot c(\Theta, \lambda)^2 \cdot t_P \cdot H \tag{11}$$

$$F_w(\Theta, \lambda) = c_w(\Theta, \lambda) \cdot \frac{\rho}{2} \cdot c(\Theta, \lambda)^2 \cdot t_P \cdot H \tag{12}$$

$$P_{Rotor}(\lambda) = n \int_0^{2\pi} (F_a(\Theta, \lambda)\ r_a - F_w(\Theta, \lambda)\ r_w)\ d\Theta\ \Omega \tag{13}$$

The resulting curve for the power coefficient has to be validated.

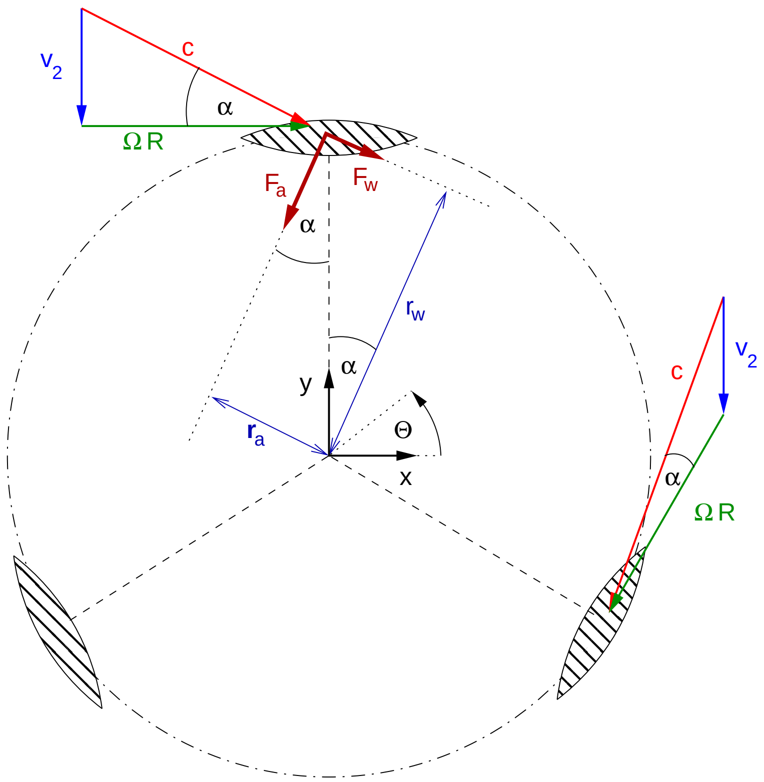

Figure 5: velocities and forces at a Darrieus-rotor

4.4 Lift Based Turbines at Low Tip Speed Ratios

At first all influences due to the angular momentum have been neglected, which are important at low tip speed ratios. Further on is this model not valid at low TSR, because of nonlinear stall effects at high angles of attack appearing at low TSR. These effect cannot be modelled analytically. So in the end this model is applicable for a general layout, but for more specific investigation a numerical model of the behaviour at low velocities is needed. experimental curves show, that the power coefficient at low TSR is infact negativ, due to these stall effects. Because of this, the machine is not able to start by itself.

But as we remember, the behaviour of the power of drag based design was different. If we could use only the drag power to start and switch to lift based operation later, a complicated starting procedure will not be needed. In order to achieve this, the blades have to be pitched for drag use. For an effectiv acceleration, the blades have to be pitched during rotation.

These kinetics and the influence [2] on the stream have benn modelled numerically. Just for starting procedures the use of the drag force is supposed to be a suitable solution.

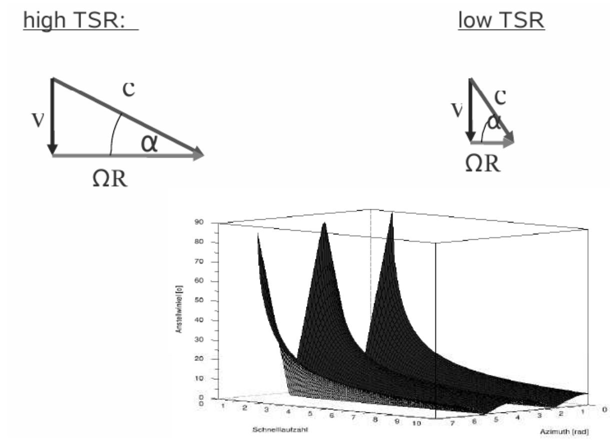

Figure 6: velocities and angle of attack at low TSR

5 Conclusions

In the paragraphs above, different possibilities for extracting energy from free stream have been shown, and analytical models have been derived. The concept of a vertical axis free stream turbine has been discussed in more detail. The analytical approaches provide valid results for higher velocities (e.g. during normal stationary operation). A weak point of the analytical model are the result at low TSR. Here the importance of exact numerical models is shown. With the help of the valid analytical reults and the additional numerical results, free stream turbines for the use in wind oder tidal curent conversion can be modelled very accurate.

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