Technische Universität München
Department of Informatics
Chair for Efficient Algorithms
Prof. Dr. Ernst W. Mayr/Dr. Jens Ernst
Johannes Nowak

Summer Term 2005
Exercise Sheet 9
June 28, 2005

Selected Topics in Computational Biology

Due: 04.07.2005 **after** *the lecture*

Exercise 1 (10 points)

Let $G = (V, E)$ be a graph and S be a minimum (edge) cut of G. S splits the graph into two induced subgraphs. Let $\bar{H}$ be the smaller of these subgraphs with $k > 1$ vertices. Prove that $|S| \leq k$ and that if $|S| = k$ then $\bar{H}$ is a clique.

Exercise 2 (10 points)

Prove the following properties of the Pearson correlation coefficient $corr_p = \frac{cov(X_i, X_j)}{\sigma_i \cdot \sigma_j}$ as defined in the lecture. For some α and β

a) $-1 \leq corr_p(X_i, X_j) \leq 1$

b) $corr_p(\alpha X_i, X_j) = corr_p(X_i, X_j)$

c) $corr_p(X_i + \beta \cdot \mathbf{1}, X_j) = corr_p(X_i, X_j)$

Exercise 3 (10 points)

For two expression profiles X_i and X_j, the mutual information $M(X_i, X_j)$ is given by

$$M(X_i, X_j) = H(X_i) - H(X_i | X_j)$$

a) Show that the mutual information does not satisfy the triangle inequality

b) The relative entropy between the joint distribution and the product distribution of X_i and X_j is

$$I(X_i, X_j) = \sum_{x \in R} \sum_{y \in R} p_{x,y} \cdot \log \frac{p_{x,y}}{p_x \cdot p_y}$$

. Show that $I(X_i, X_j) = M(X_i, X_j)$