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Randomisierte Algorithmen

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Aufgabe 1

Satz 6.7 aus dem Skript:
Für jedes System von einfachen Wegen mit Congestion C und Dilation D gibt es ein Strategie, die Pakete so zu verschicken, dass in $O(C + D)$ Zeit alle Pakete ihr Ziel erreicht haben.

I will give a general outline of the proof and ask you to complete sections for the exersize.

Proof. Let $I_0 = max(C, D)$ and $I_j = \log I_{j-1}$ for all $j \geq 1$. The first step is to assign an initial delay to each packet, chosen independently and uniformly at random from the range $\Delta_1 = [C]$. In the resulting schedule, S_1, a packet that is assigned a delay of δ waits in its source node for δ steps, then moves on without waiting until it reaches its destination. The length of S_1 is at most $D + C$. We use Lovász Local Lemma to show that if the delays are chosen independently and uniformly at random and I_1 is sufficiently large, then with nonzero probability the congestion at any edge in any I_1^3-interval is at most $C_1 = (1 + \Theta(\frac{1}{I_1}))I_1^3$.

A. *Please use Lovász Local Lemma to show that if the delays are chosen independently and uniformly at random and I_1 is sufficiently large, then with nonzero probability the congestion at any edge in any I_1^3-interval is at most $C_1 = (1 + \Theta(\frac{1}{I_1}))I_1^3$. (Hint: Associate a bad event with each edge.)*

We now break schedule S_1 into I_1^4-frames and continue to schedule each frame independently. So each frame can be viewed as a seperate scheduling problem where the origin of a packet is its location at the beginniing of the frame, and it destination is it location at the end of the frame. Our next refinement step will be to choose, for each frame, a random initial delay for each packet that visits at least one edge within this frame. In the resulting schedule, S_2, the frames (enlarged by their delay ranges) are executed one after the otherin a way that a packet that is assigned a delay δ in some frame F waits at its first edge in F for (additional) δ steps, and then moves on without waiting until it traverses its last edge in F.
Let us concentrate in the following on some fixed frame F. Let each packet choose an additional delay out of the range $\Delta_2 = [I_1^3 - I_2^3]$. Hence the length of the resulting schedule for this frame is at most $I_1^4 + I_1^3$. We use Lovász Local Lemma to show that if the delays

are chosen independently and uniformly at random and I_2 is sufficiently large, then with nonzero probability the congestion in any I_2^3-interval is at most

$$C_2 = \left(1 + \Theta\left(\frac{1}{I_1}\right)\right)\left(1 + \Theta\left(\frac{1}{I_1}\right)\right)\left(\frac{1}{1 - (\frac{I_2}{I_1})^3}\right) I_2^3$$

B. *Please use Lovász Local Lemma to show that if the delays are chosen independently and uniformly at random and I_2 is sufficiently large, then with nonzero probability the congestion in any I_2^3-interval is at most*

$$C_2 = \left(1 + \Theta\left(\frac{1}{I_1}\right)\right)\left(1 + \Theta\left(\frac{1}{I_1}\right)\right)\left(\frac{1}{1 - (\frac{I_2}{I_1})^3}\right) I_2^3$$

(Hint: Associate a bad event with each edge.)

We continue to refine each I_2^4-frame in F recursively until we reach a round k, in which $I_k = \Theta(1)$. We end up with a schedule S_k with total length at most

$$(D + C)\prod_{j=1}^{k-1}\left(1 + \frac{1}{I_j}\right) \leq 1.1(D + C) = O(D + C)$$

and with a congestion C_k in each I_k^3-interval of at most

$$I_k^3\left(1 + \frac{3}{I_1}\right)\prod_{j=1}^{k-1}\left(1 + \frac{4}{I_{j+1}}\right)\left(\frac{1}{1 - (\frac{I_{j+1}}{I_j})^3}\right) \leq 2.5 I_k^3 = \Theta(1)$$

Since the total length of S_k is $O(C + D)$ and each interval has congestion $\Theta(1)$, then a schedule with overall runtime $O(C + D)$ exists.
□