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Effiziente Algorithmen und Datenstrukturen I

Aufgabe 1 (10 Punkte)

Given the key of an element x in an n-node binary search tree (choose a BST with suitable properties) and a natural number i, show how to augment the tree to find the i-th successor of x in the linear order of the tree in $O(\log n)$ time.

Aufgabe 2 (10 Punkte)

Suppose that we wish to keep track of a *point of maximum overlap* in a set of itervals - a point that has the largest number of intervals in the set of intervals overlapping it.

1. Show that there will always be a point of maximum overlap which is an endpoint of one of the segments.
2. Design a data structure that efficiently supports the operations INSERT, DELETE, and FIND_POM which are defined as follows:
   (a) INSERT(i, j): Inserts the interval $[i, j]$ in the set of intervals.
   (b) DELETE(i, j): Deletes the interval $[i, j]$ from the set of intervals.
   (c) FIND_POM: Returns a point of maximum overlap.

   (*Hint:* Keep a red-black tree of all the endpoints. Associate a value of $+1$ with each left endpoint, and associate a value of -1 with each right endpoint. Augment each node of the tree with some extra information to maintain the point of maximum overlap.)

Aufgabe 3 (10 Punkte)

The mean M of a set of k integers $\{x_1, x_2, \ldots, x_k\}$ is defined as

$$M = \frac{1}{k} \sum_{i=1}^{k} x_i.$$

We want to maintain a data structure D on a set of integers under the normal INIT, INSERT, DELETE, and FIND operations, as well as a MEAN operation, defined as follows:

1. INIT(D): Create an empty structure D.
2. INSERT(D, x): Insert x in D.

3. DELETE(D, x): Delete x from D.

4. FIND(D, x): Return pointer to x in D.

5. MEAN(D, a, b): Return the mean of the set consisting of elements x in D with $a \leq x \leq b$.

Describe how to modify a standard red-black tree in order to implement D, such that INIT is supported in $O(1)$ time and INSERT, DELETE, FIND, and MEAN are supported in $O(\log n)$ time.