Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen
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Winter Semester 2012/13
Problem Set 2
October 30, 2012

Parallel Algorithms

Due Date: November 6, 2012 **before** *class!*

Problem 1 (10 Points)

Consider an array with n distinct elements. The task is to search the array for a particular element x. We know that a sequential algorithm will always have a worst-case running time of n steps. Derive an efficient parallel algorithm for this task on an EREW (exclusive read / exclusive write) PRAM with $p \leq n$ processors.

Problem 2 (10 Points)

Given a boolean function F, show that

$$\mathbf{size}(F) \leq 2^{\mathbf{depth}(F)},$$

where $\mathbf{size}(F)$ denotes the size of a minimal circuit with outdegree 1 which computes F, and $\mathbf{depth}(F)$ its depth, respectively.

Problem 3 (10 Points)

Every d-ary tree $G = (V, E)$ contains a vertex v such that the size of the subtree with root v is at least $\frac{1}{d+1}|V|$ and at most $\frac{d}{d+1}|V| + 1$.

Problem 4 (10 Points)

Given a boolean function F, show that

$$\mathbf{depth}(F) \leq \alpha \log \mathbf{size}(F),$$

where $\alpha = 2 \cdot \log(3/2)^{-1}$.
Hint: Use Problem 3.