Parallel Algorithms

Due Date: November 20, 2012 **before** *class!*

Problem 1 (10 Points)

Recall: A sequence is *bitonic* if there is an integer $k \in \{0, \ldots, n\}$ such that
$x_1 \geq \cdots \geq x_k \leq \cdots \leq x_n$. Show that a bitonic sorting network can be constructed as
follows:

- sort $(x_1, x_3, x_5, \ldots)$ and $(x_2, x_4, x_6, \ldots)$ in bitonic sorters whose lines are interleaved,

- compare and interchange the outputs in pairs, beginning with the least significant pairs.

Problem 2 (10 Points)

Use the zero-one principle (the 0/1 sorting lemma) to show that the bitonic sorting network
sorts bitonic sequences correctly.

Problem 3 (10 Points)

Consider a bitonic sequence $X = (x_0, \ldots, x_{n-1})$ of length $n = 2^k, k \in \mathbb{N}$. Assume without
loss of generality that all the elements in the sequence are distinct. Furthermore, define

$$
\begin{aligned}
L(X) &= \left(\min\{x_0, x_{n/2}\}, \min\{x_1, x_{n/2+1}\}, \ldots, \min\{x_{n/2-1}, x_{n-1}\} \right), \\
R(X) &= \left(\max\{x_0, x_{n/2}\}, \max\{x_1, x_{n/2+1}\}, \ldots, \max\{x_{n/2-1}, x_{n-1}\} \right).
\end{aligned}
$$

(i) Show that the horizontal cut with the unique cross-over property can be determined
in at most k comparisons.

(ii) Deduce that $L(X)$ and $R(X)$ can be determined in at most k comparisons.

(iii) Show that the results of the comparisons made by the bitonic sorting network are
uniquely determined by the results of $2n - k - 2$ comparisons.

Problem 4 (10 Points)

Draw a bitonic sorting network for $n = 16$. Use both variants of drawing sorting networks
you have seen, namely

(i) a comparator is drawn as a single line between the two wires,

(ii) a comparator is drawn as a gate having two inputs and two outputs.

Please always state in which direction your comparators work (i.e. if the smaller element
is to be on the bottom or on the top).