Traveling Salesman

Given a set of cities ($\{1,\ldots,n\}$) and a symmetric matrix $C = (c_{ij})$, $c_{ij} \geq 0$ that specifies for every pair $(i,j) \in [n] \times [n]$ the cost for travelling from city i to city j. Find a permutation π of the cities such that the round-trip cost

$$c_{\pi(1)\pi(n)} + \sum_{i=1}^{n-1} c_{\pi(i)\pi(i+1)}$$

is minimized.

Traveling Salesman

Theorem 2

There does not exist an $O(2^n)$-approximation algorithm for TSP.

Hamiltonian Cycle:
For a given undirected graph $G = (V, E)$ decide whether there exists a simple cycle that contains all nodes in G.

- Given an instance to HAMPATH we create an instance for TSP.
- If $(i, j) \in E$ then set c_{ij} to $n2^n$ otherwise set c_{ij} to 1. This instance has polynomial size.
- There exists a Hamiltonian Path iff there exists a tour with cost n. Otw. any tour has cost strictly larger than n.
- An $O(2^n)$-approximation algorithm could decide on these cases. Hence, cannot exist unless $P = NP$.

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Metric Traveling Salesman

In the metric version we assume for every triple $i, j, k \in \{1, \ldots, n\}$

$$c_{ij} \leq c_{ij} + c_{jk} \ .$$

It is convenient to view the input as a complete undirected graph $G = (V, E)$, where c_{ij} for an edge (i, j) defines the distance between nodes i and j.

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TSP: Lower Bound I

Lemma 3

The cost $\mathrm{OPT}_{TSP}(G)$ *of an optimum traveling salesman tour is at least as large as the weight* $\mathrm{OPT}_{MST}(G)$ *of a minimum spanning tree in* G*.*

Proof:

- Take the optimum TSP tour.
- Delete one edge.
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TSP: Greedy Algorithm

- Start with a tour on a subset S containing a single node.
- Take the node v closest to S. Add it S and expand the existing tour on S to include v.
- Repeat until all nodes have been processed.

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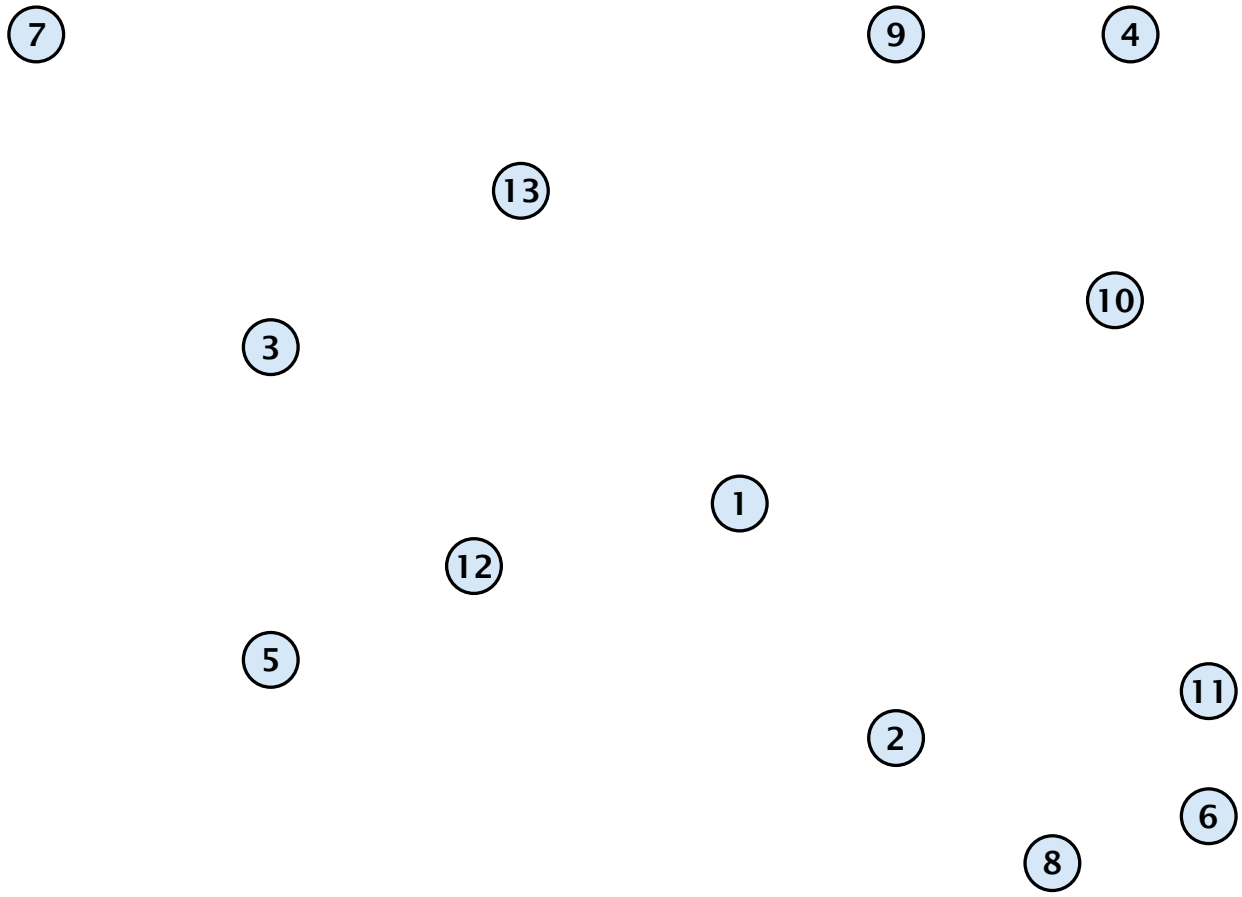

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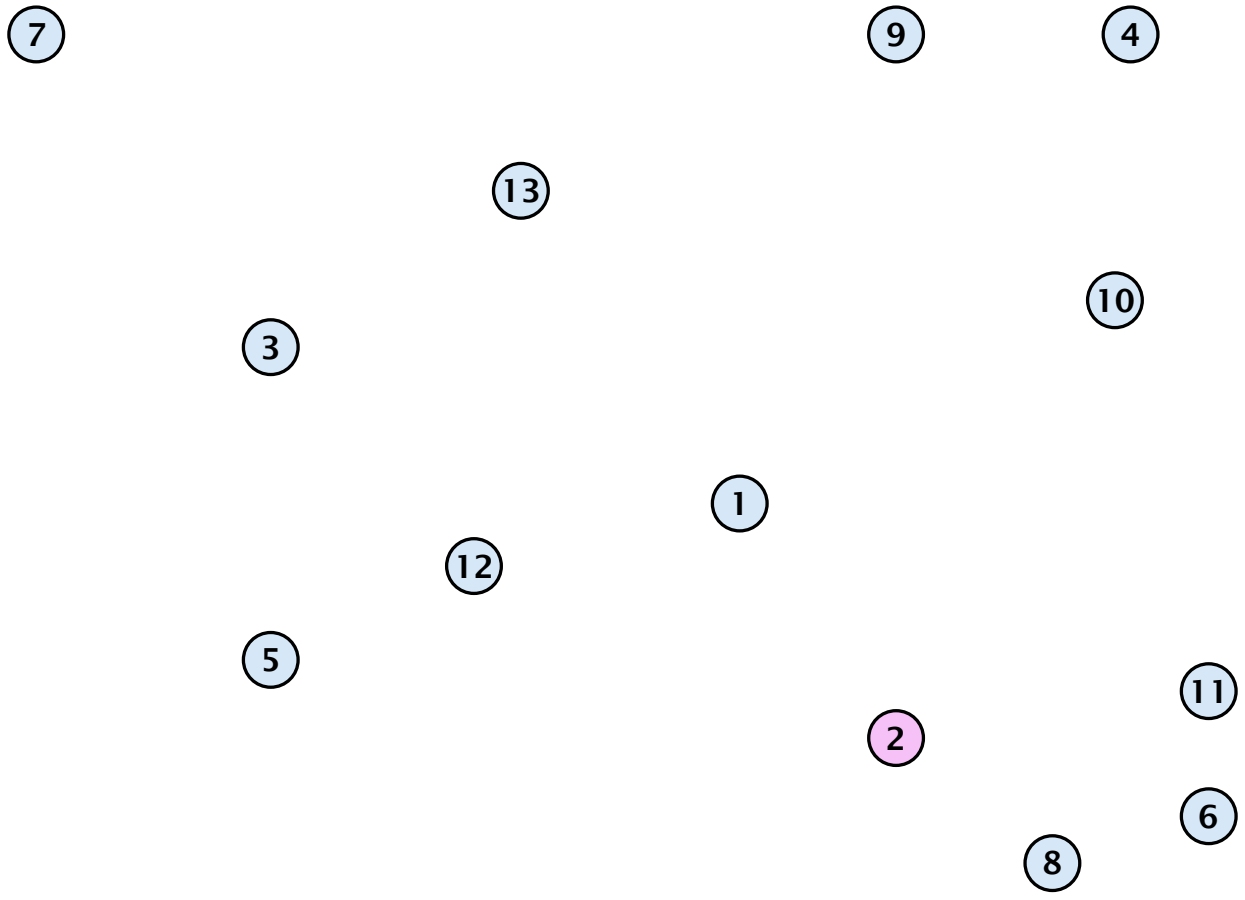

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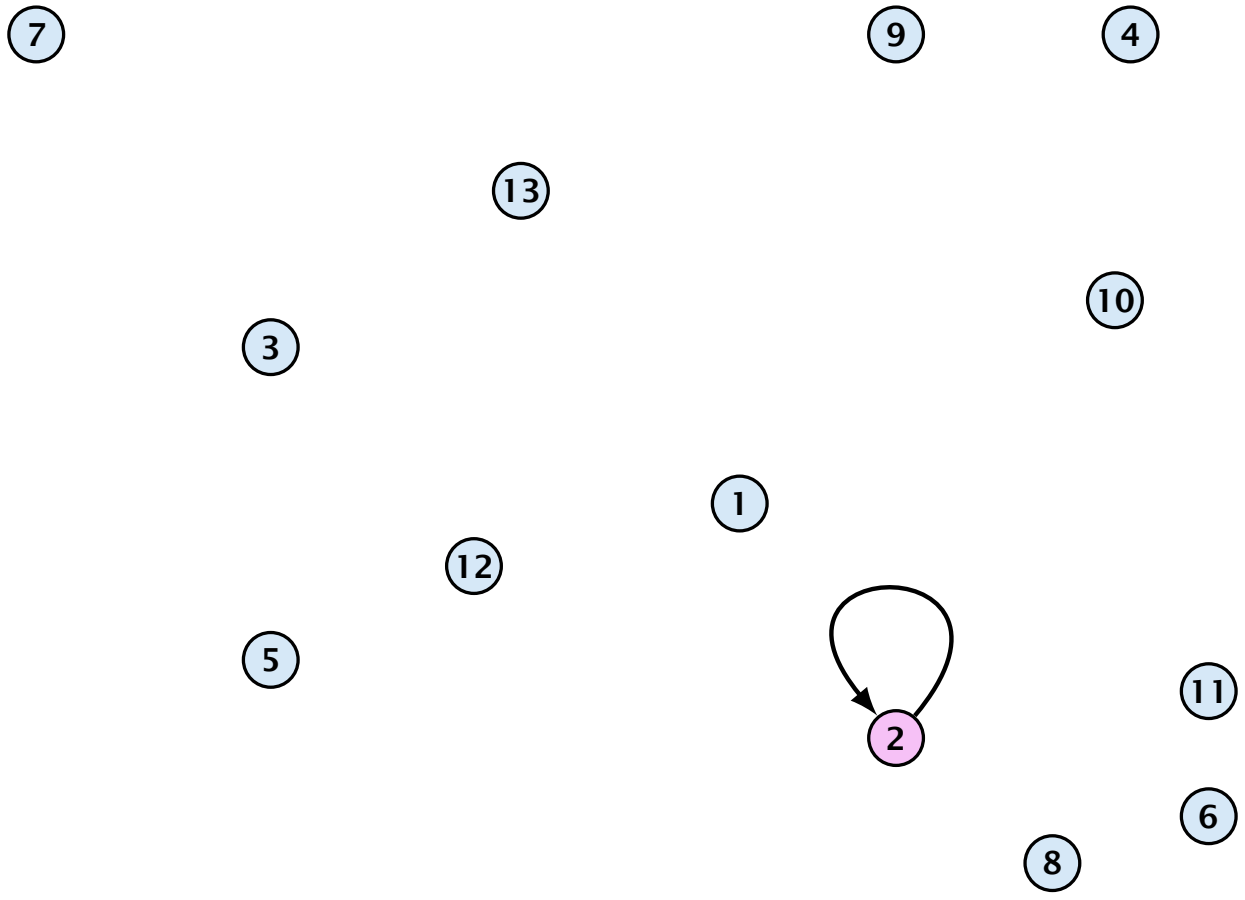

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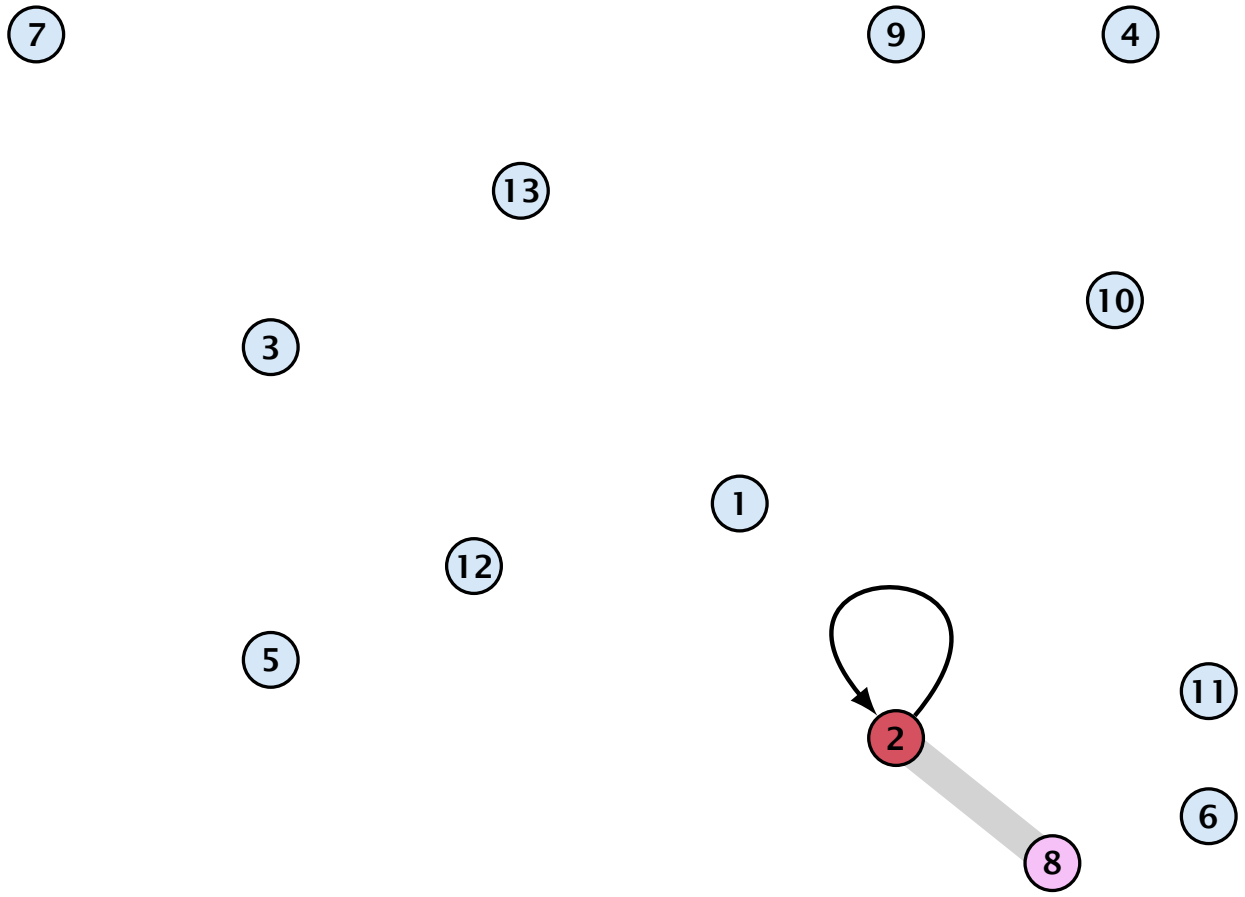

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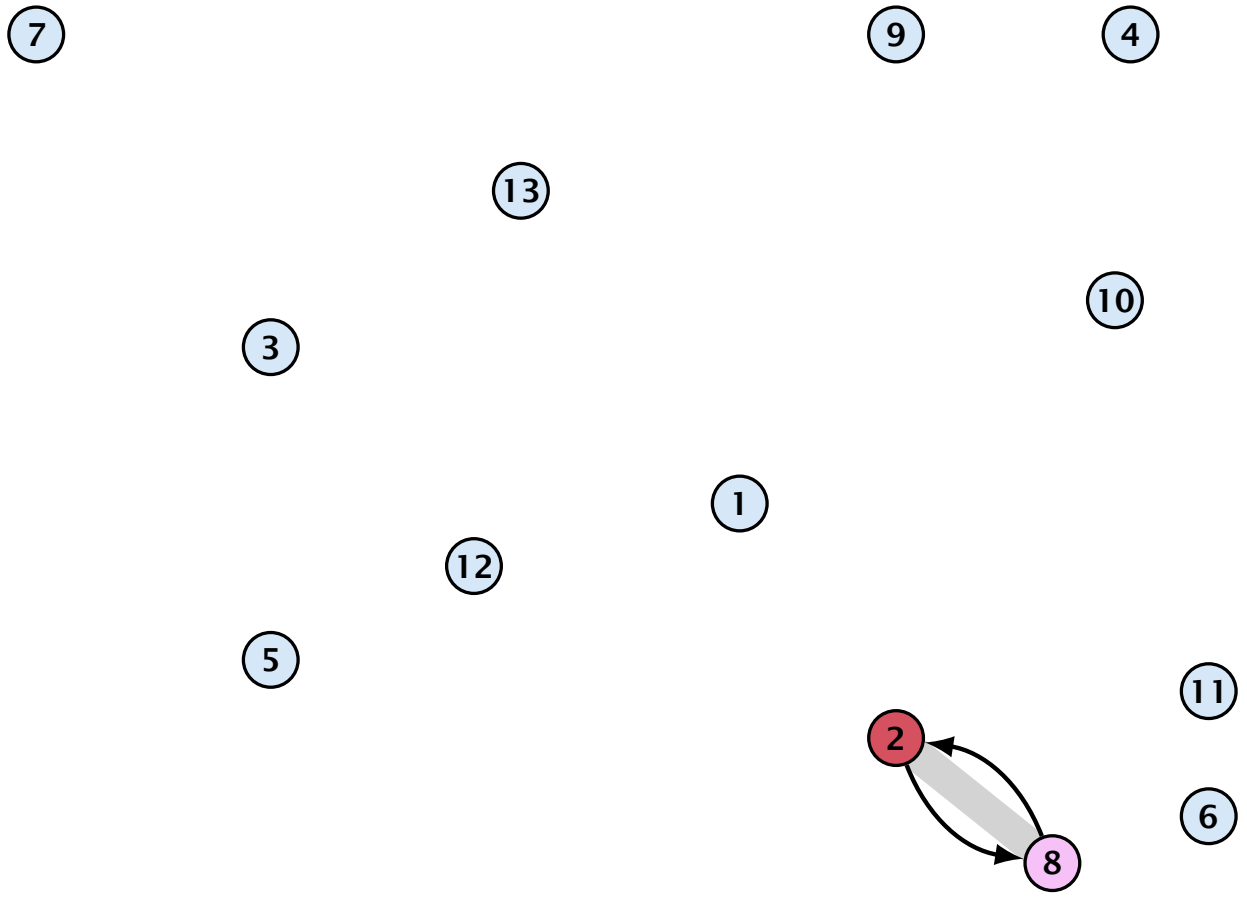

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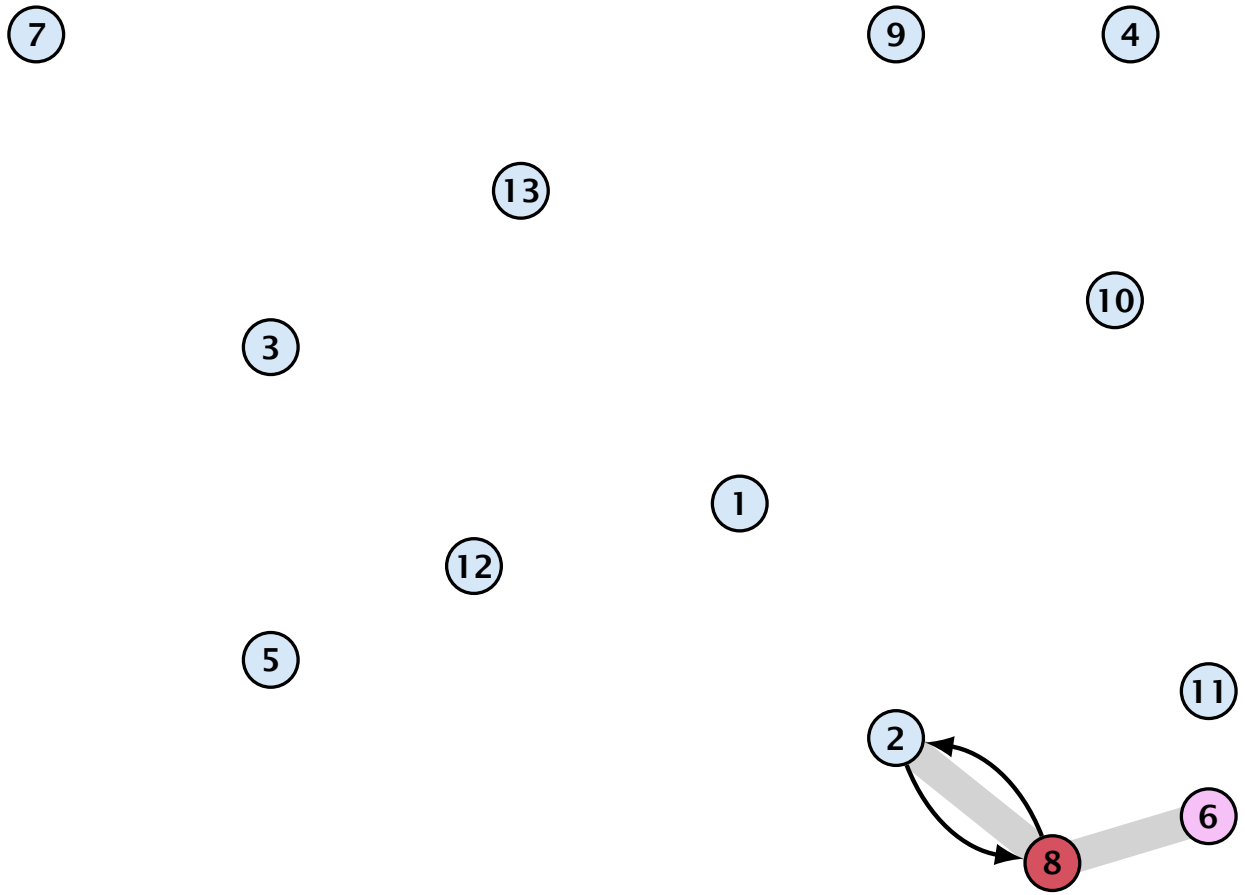

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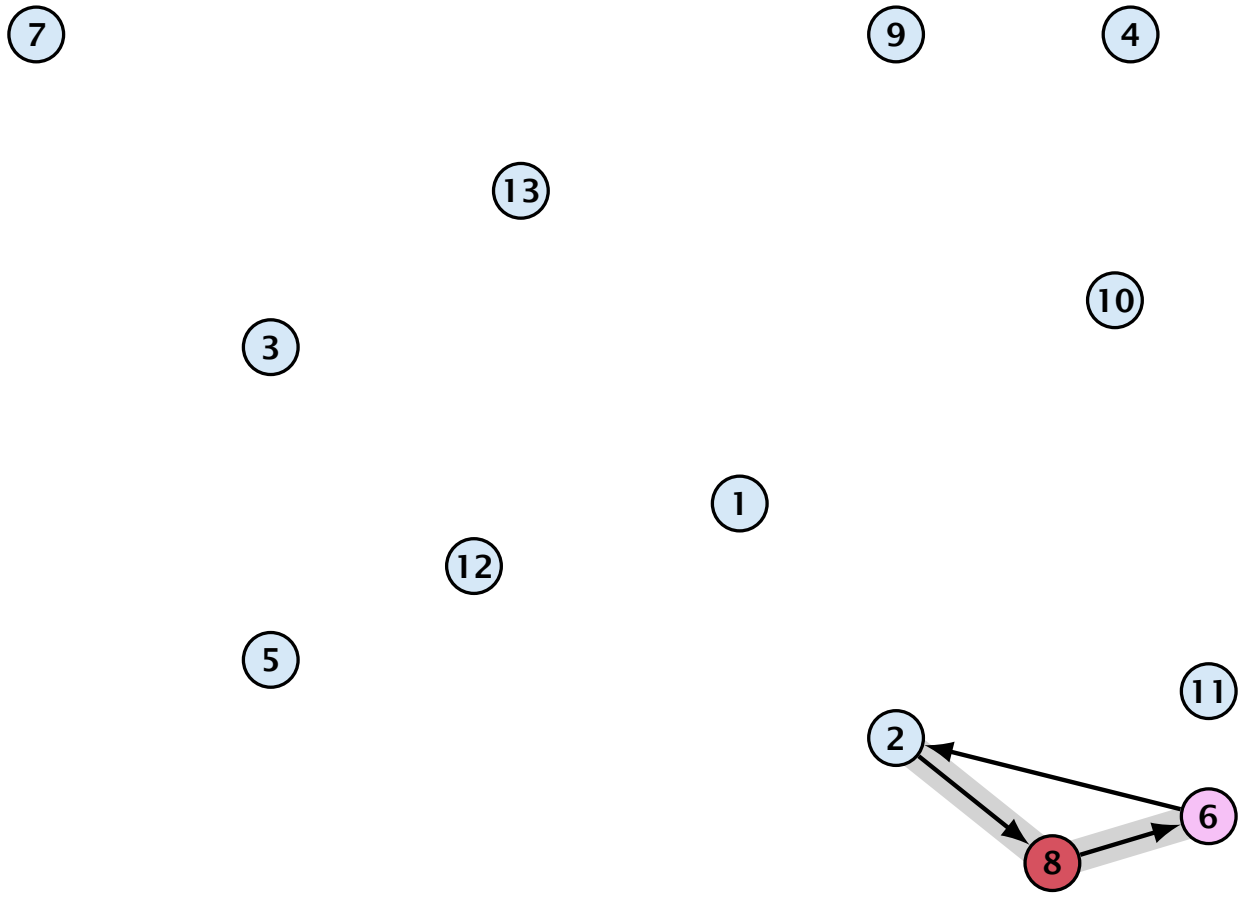

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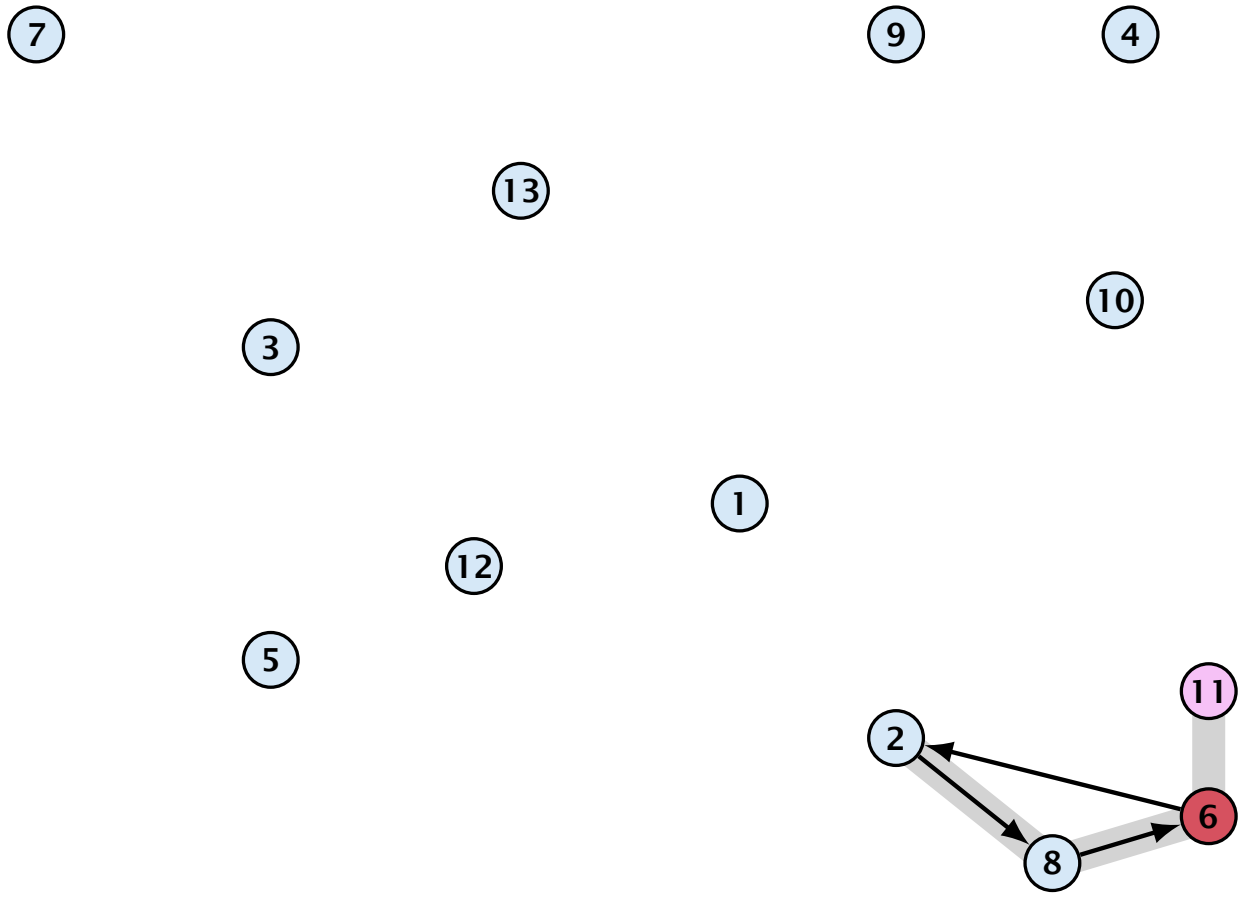

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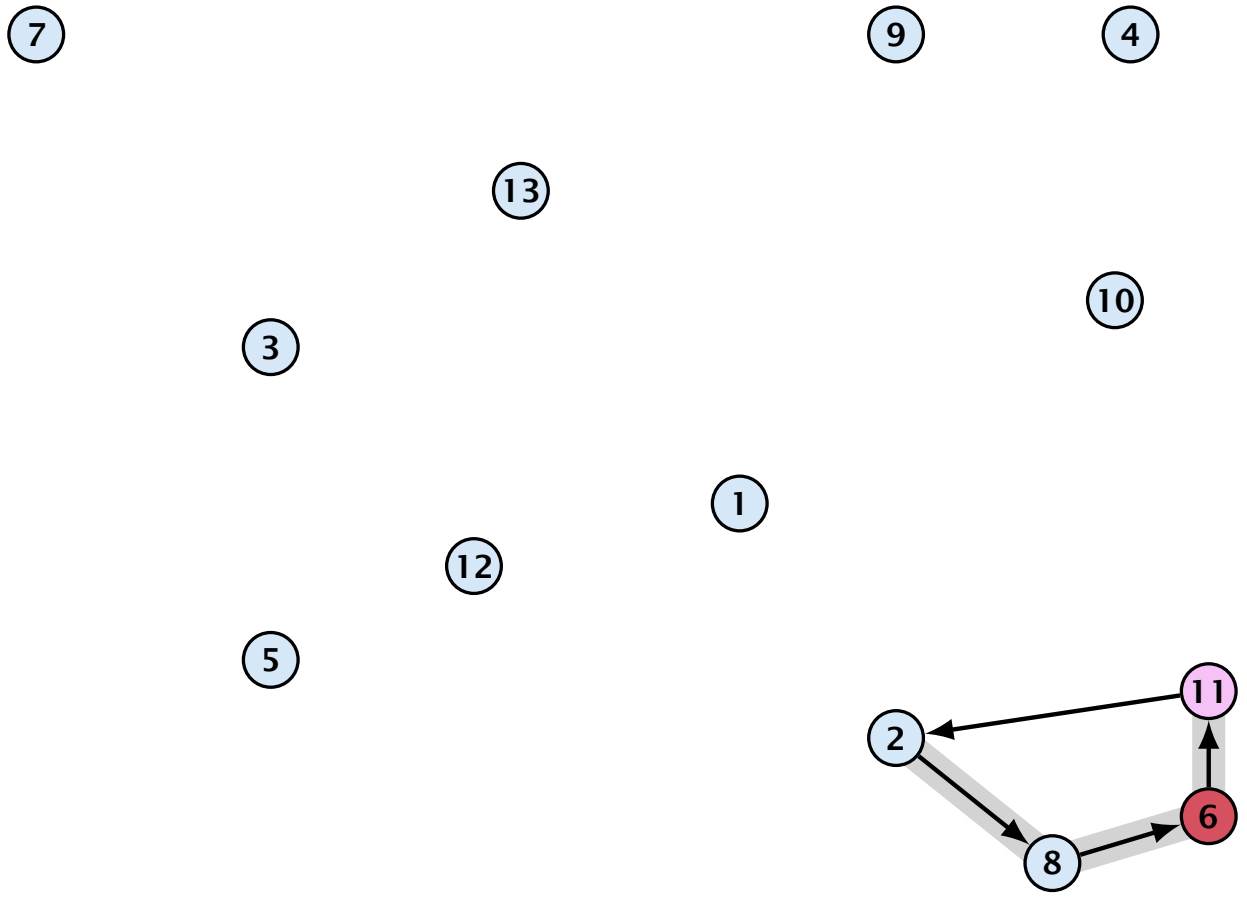

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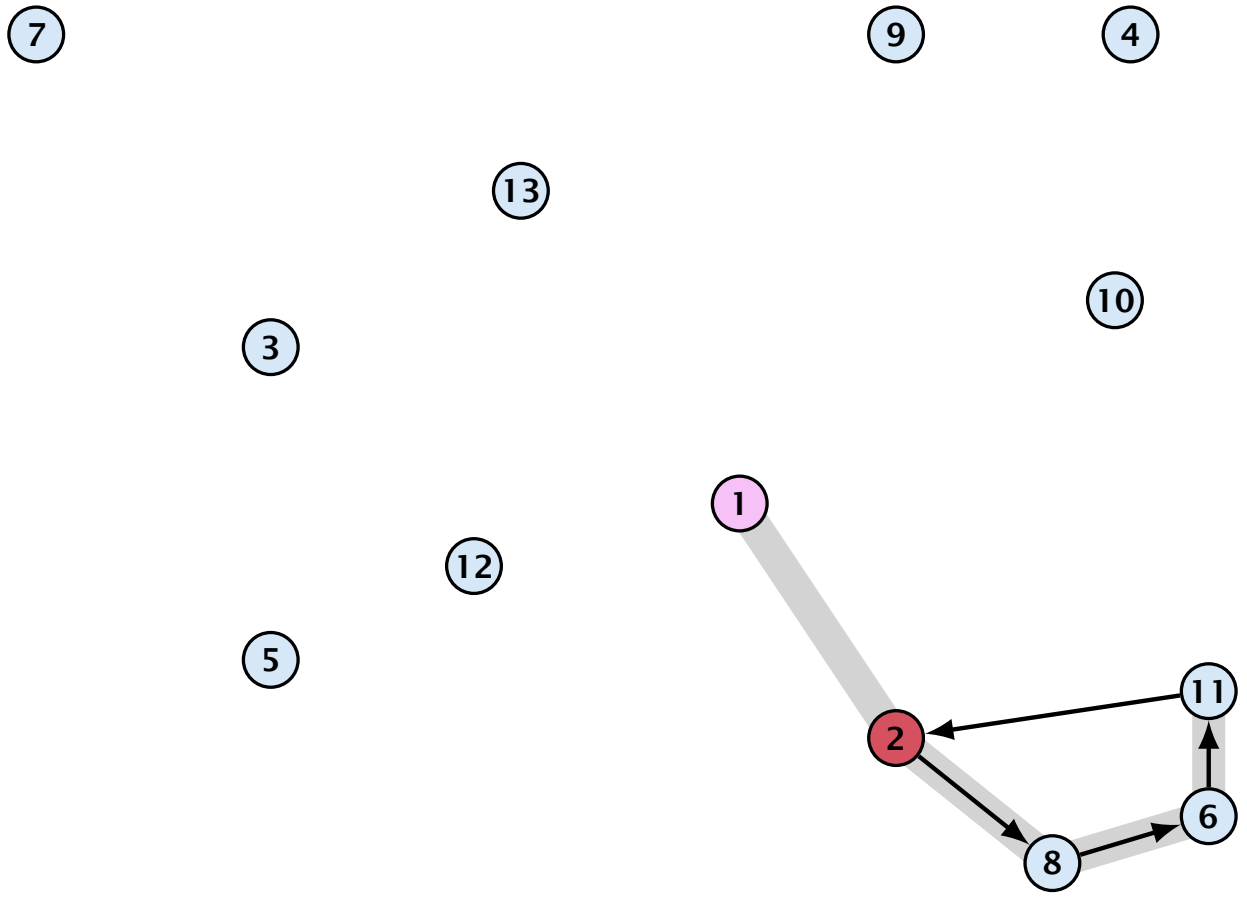

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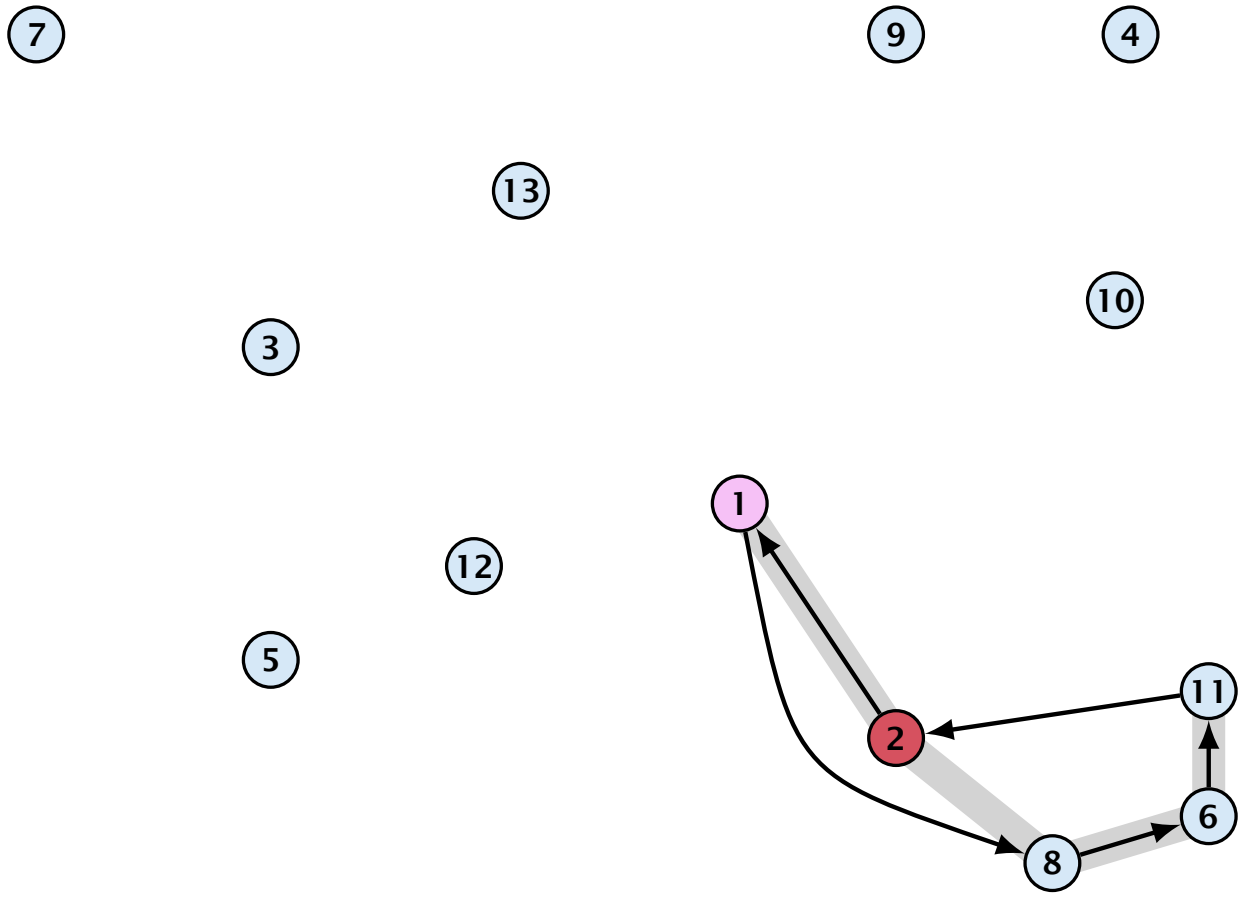

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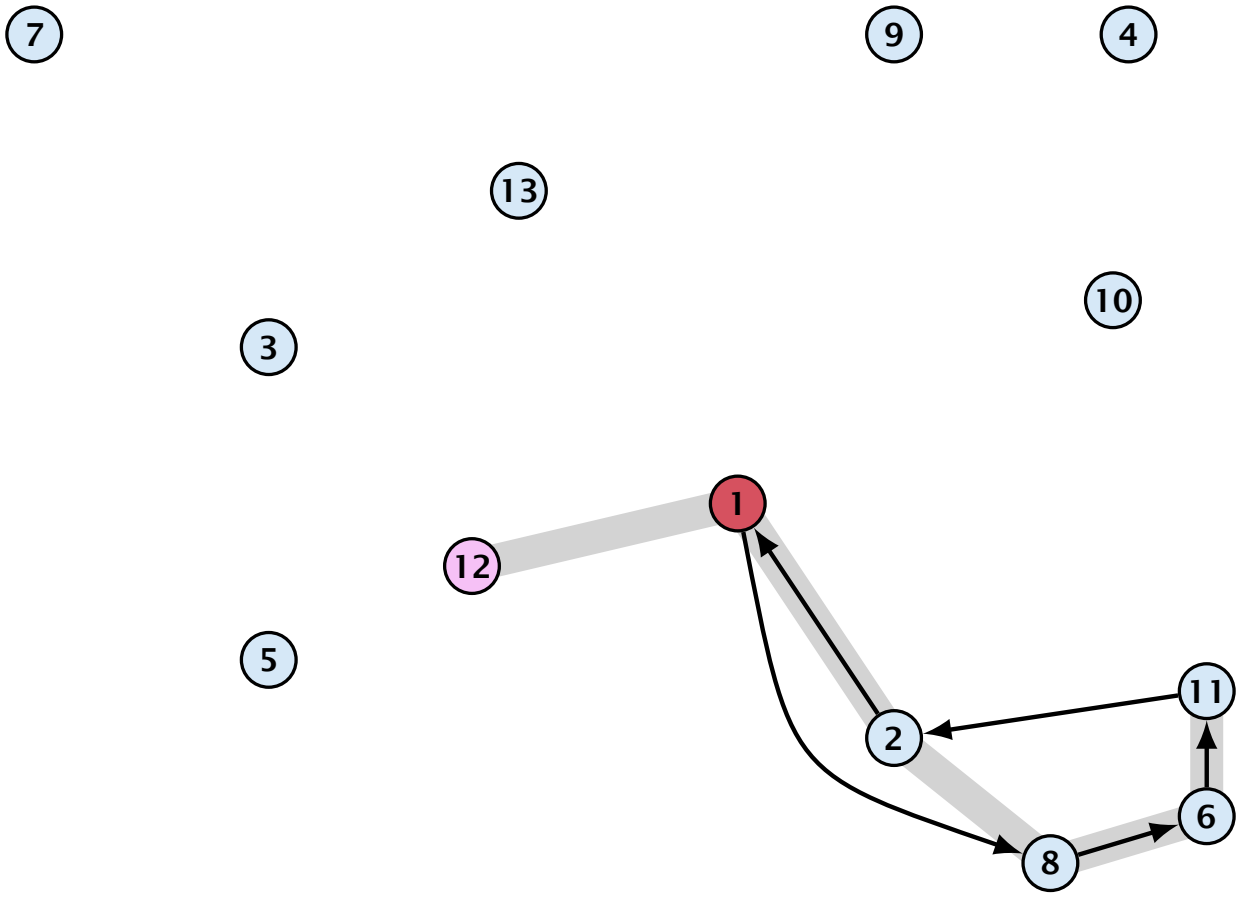

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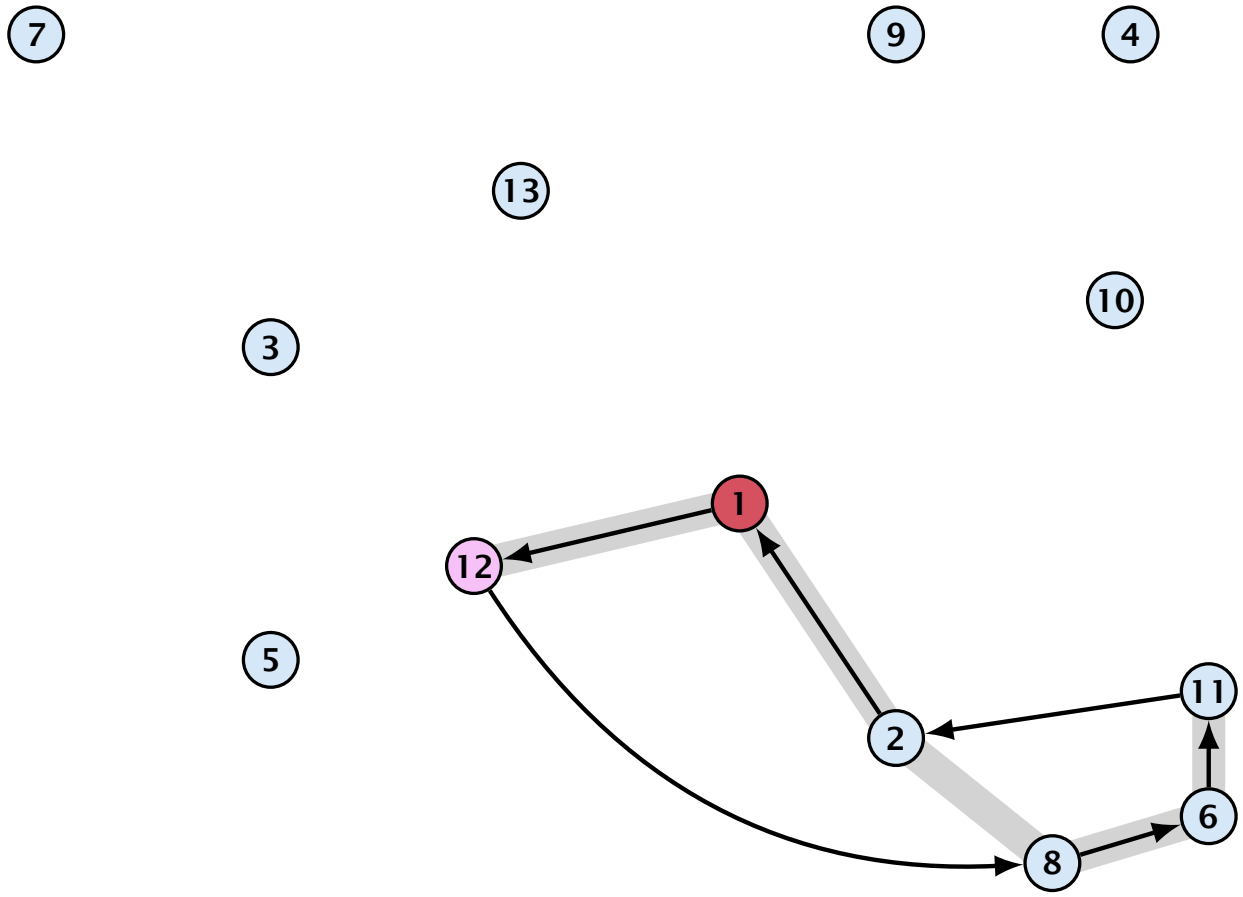

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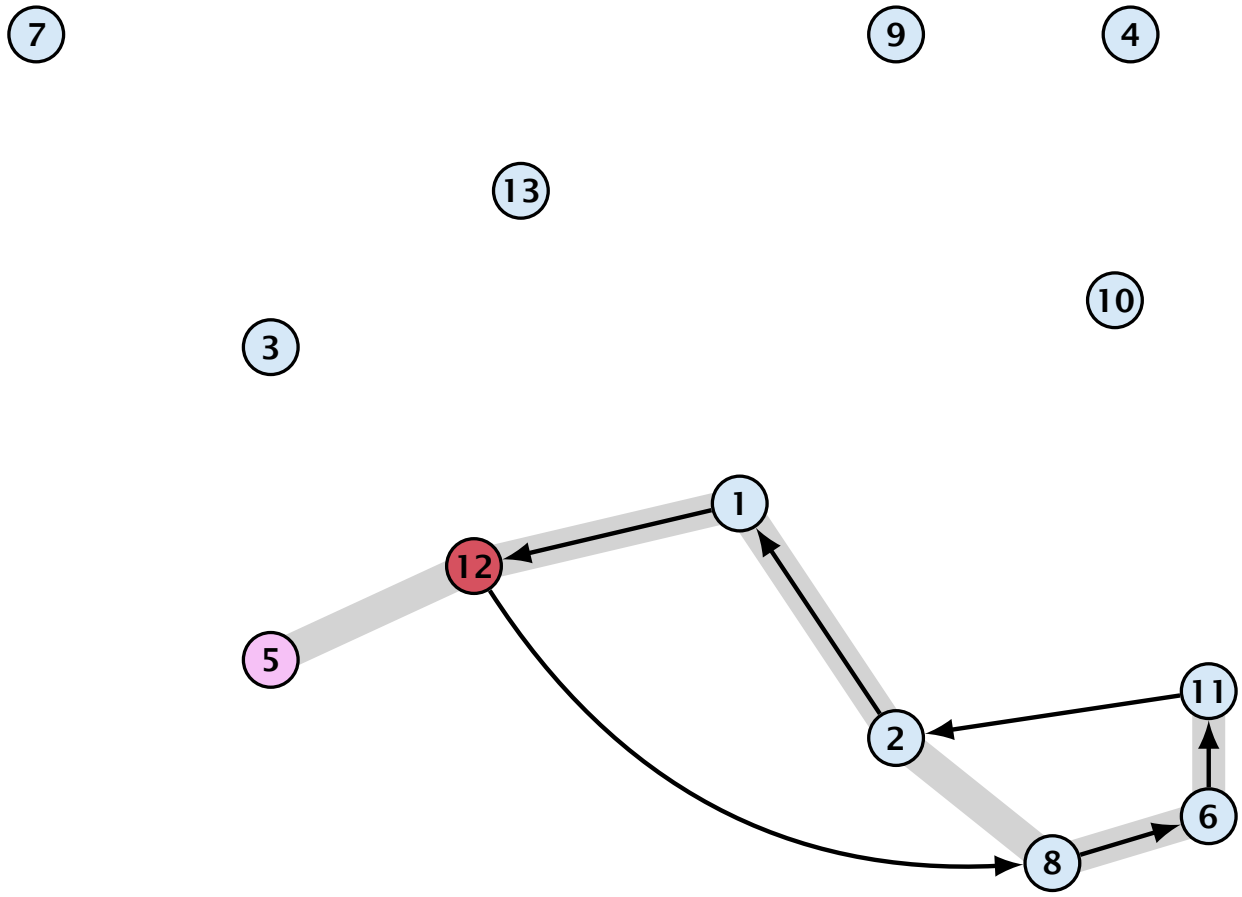

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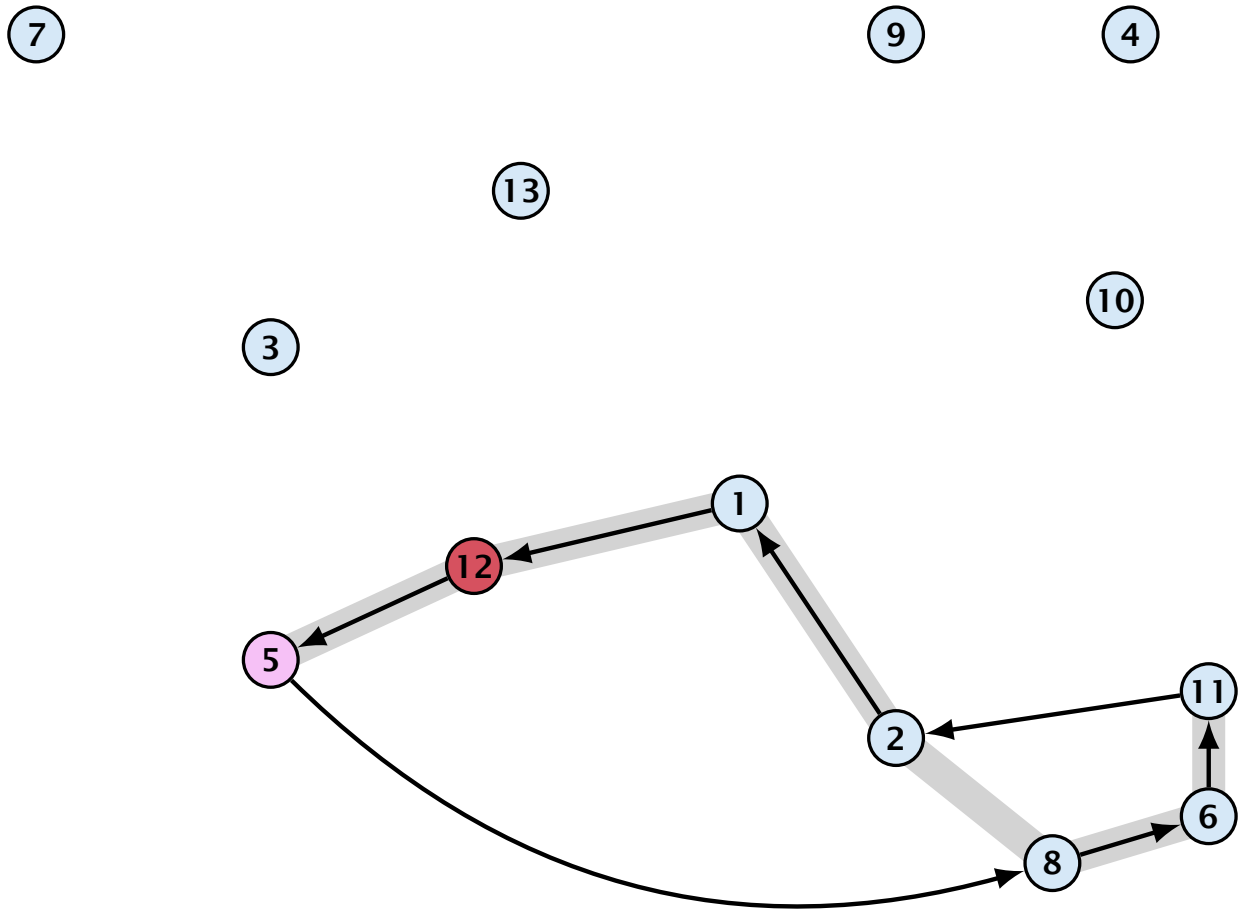

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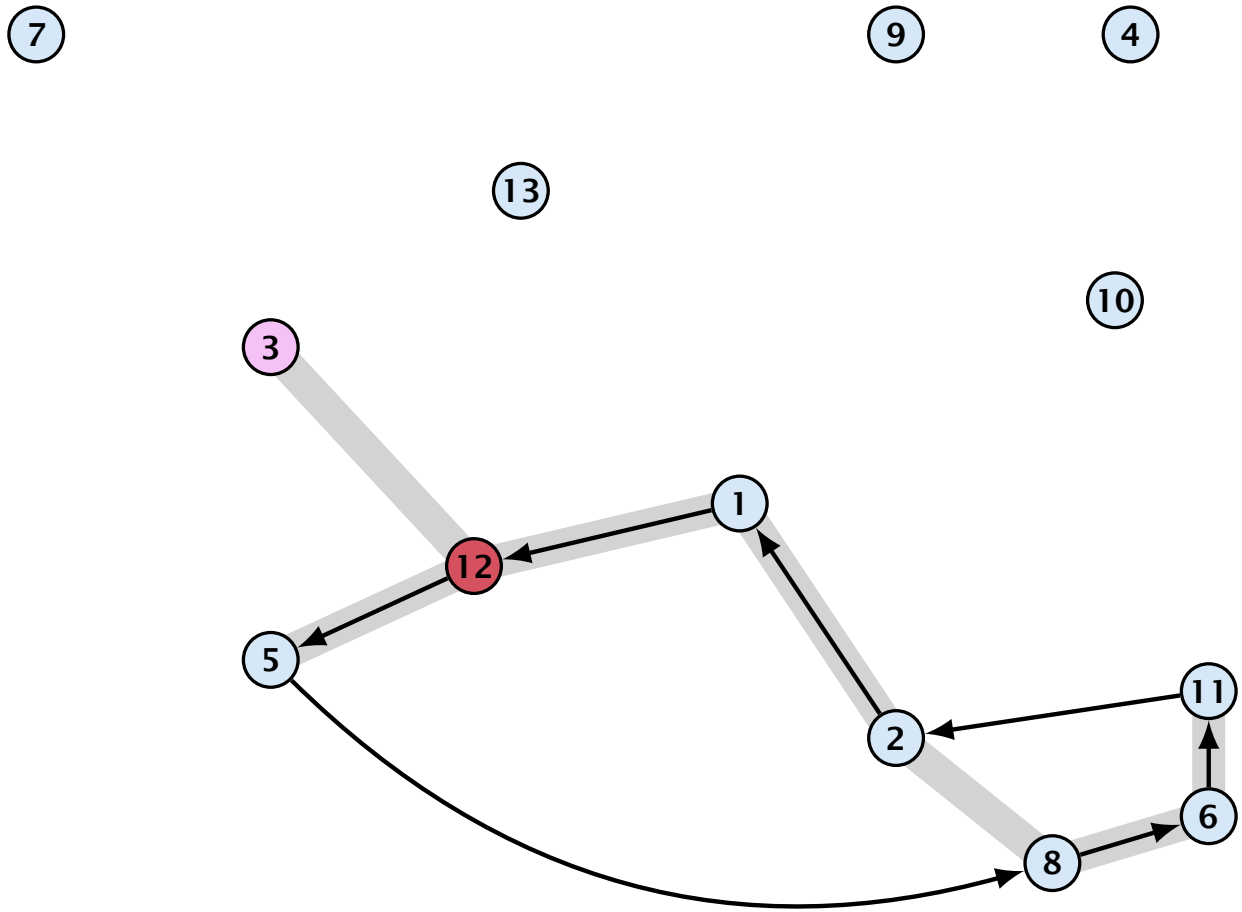

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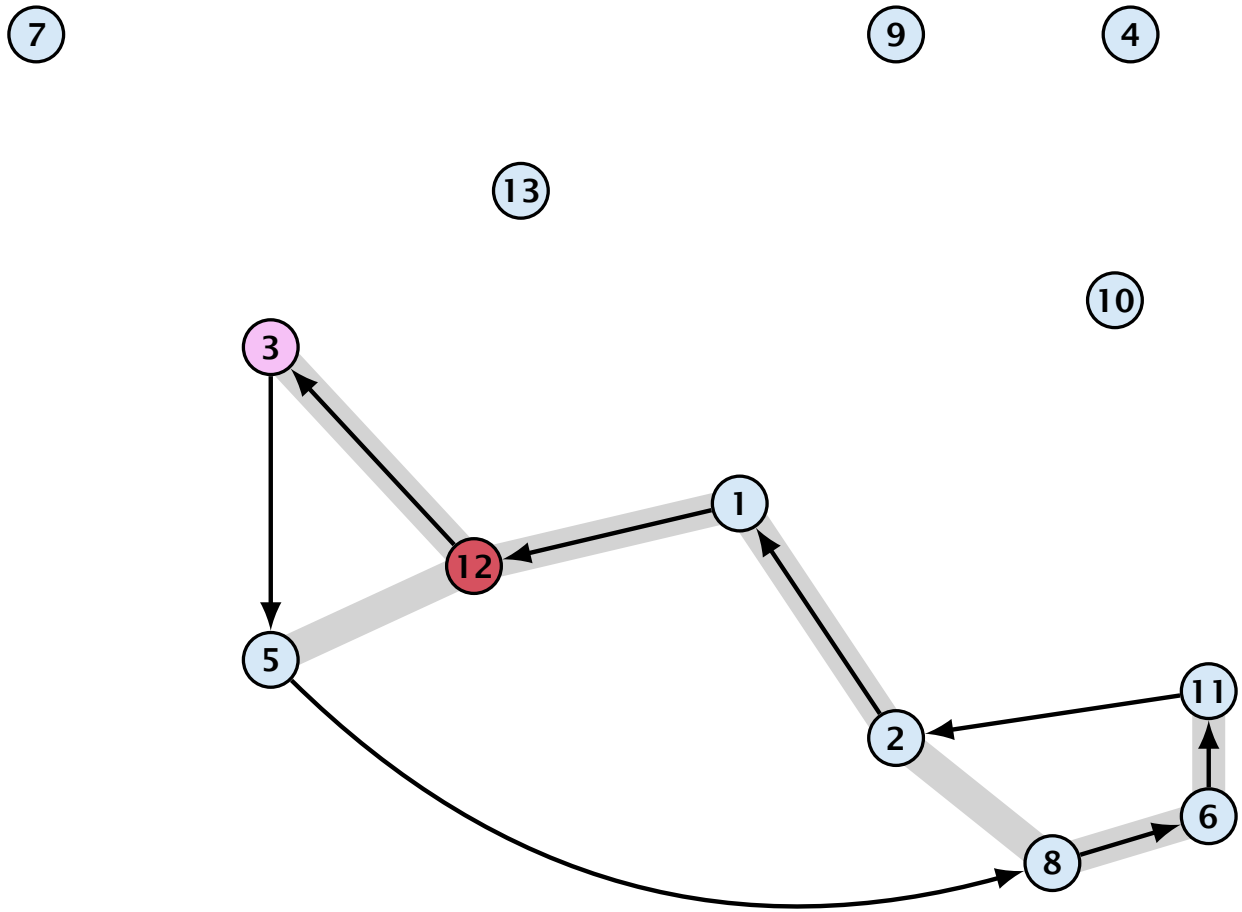

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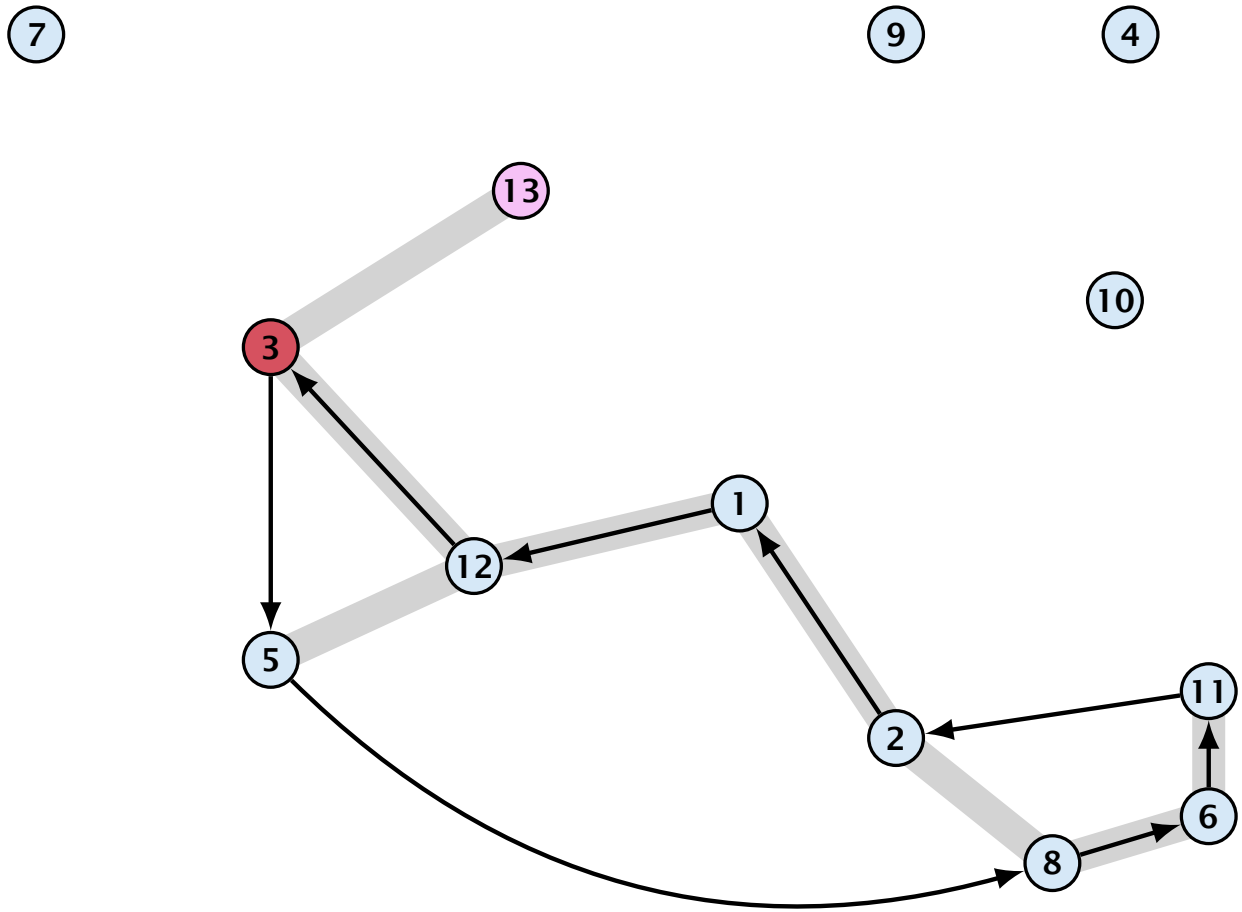

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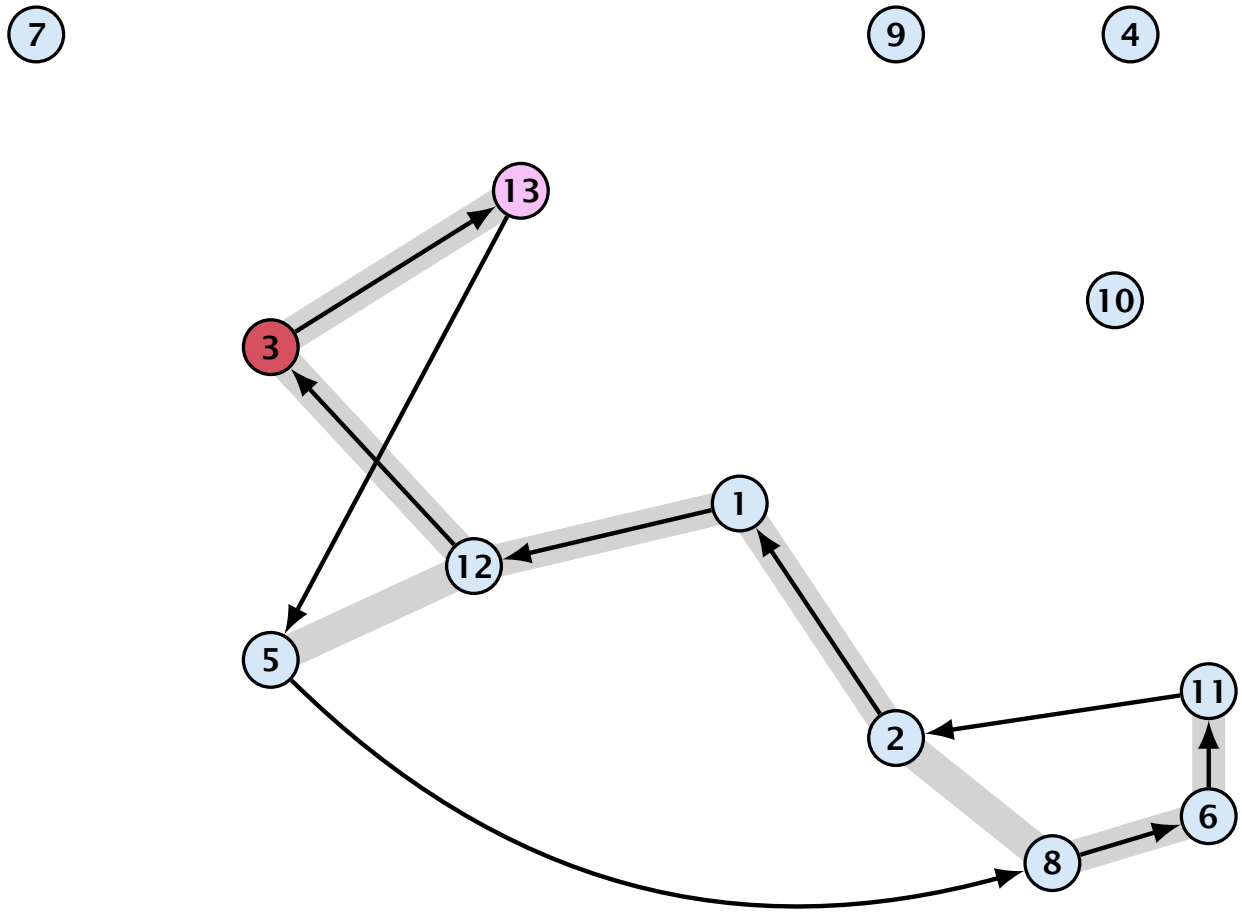

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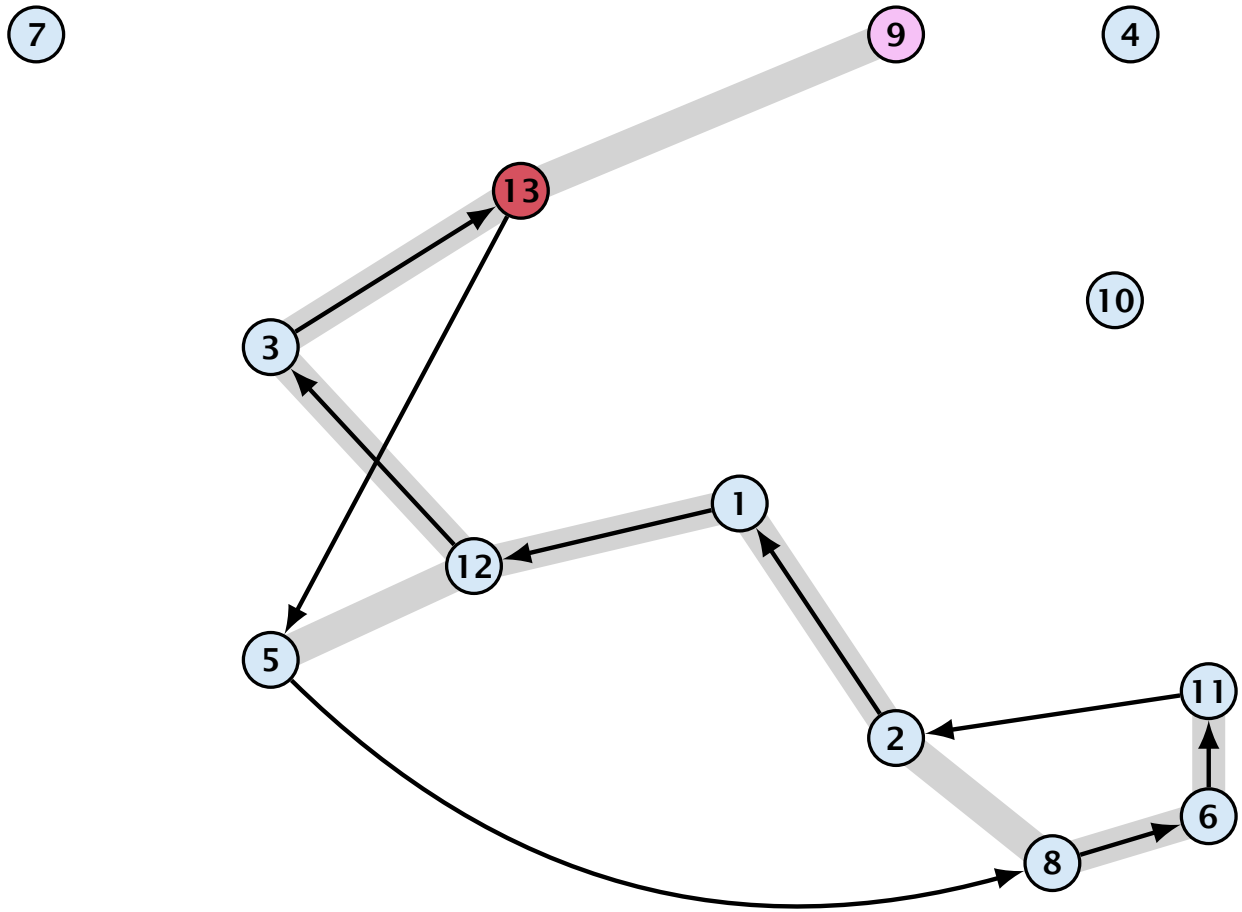

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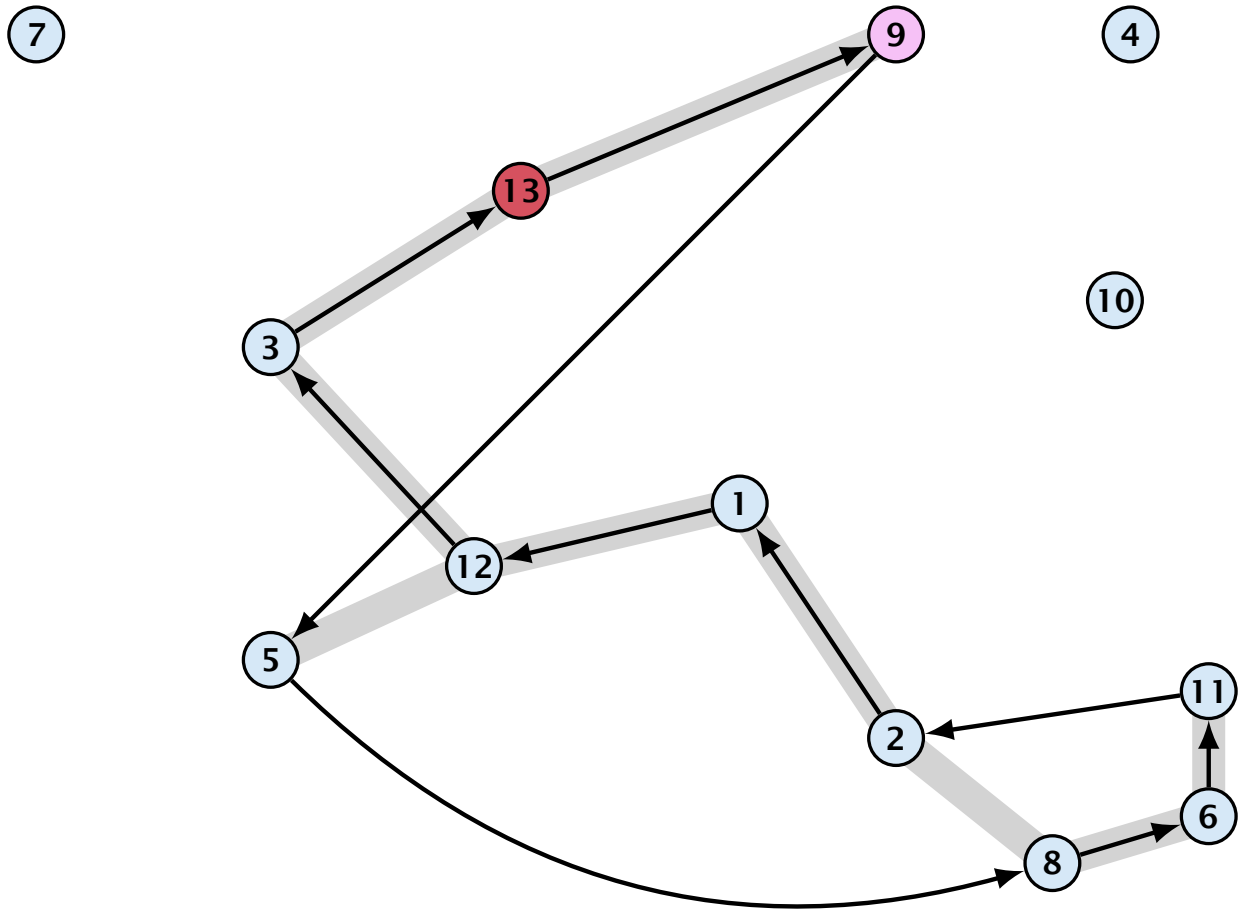

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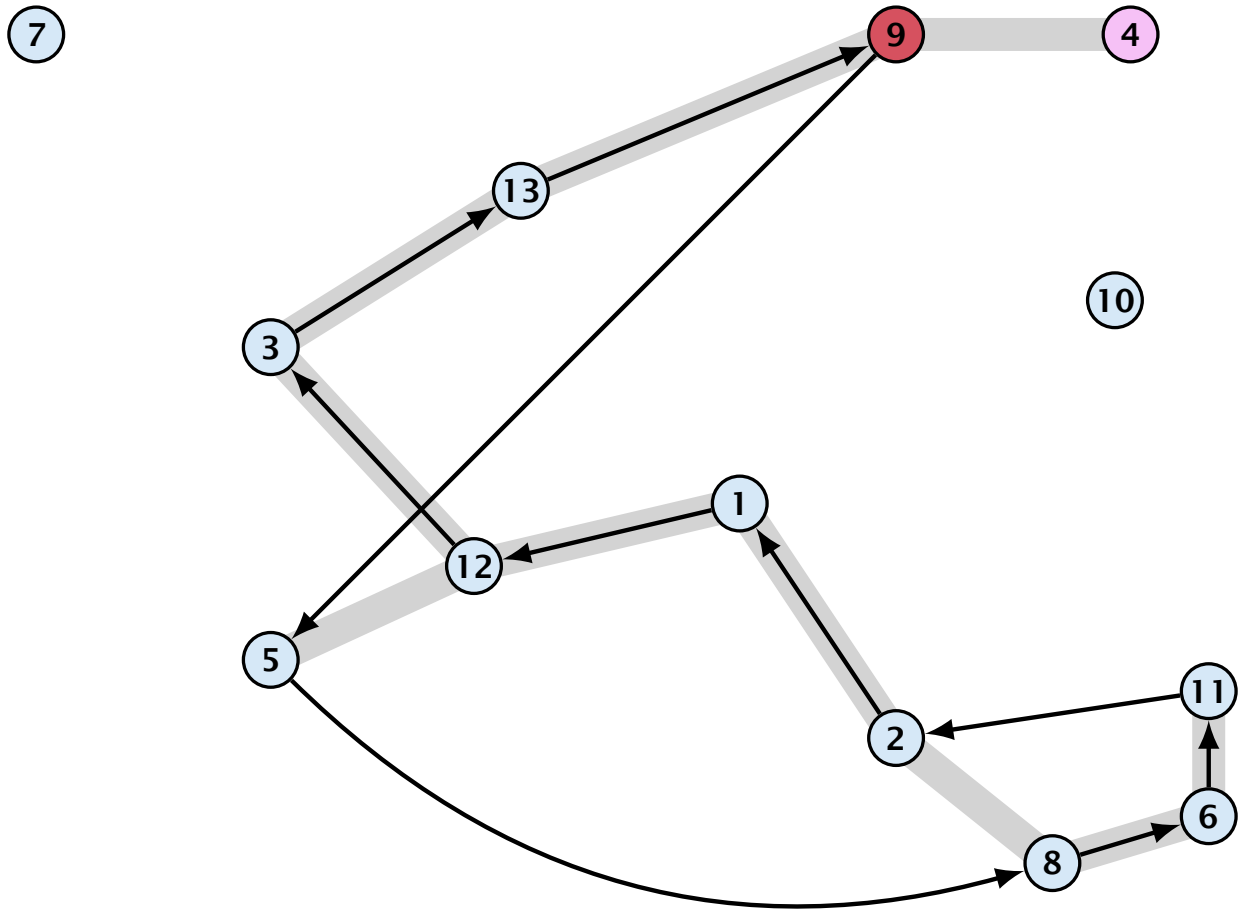

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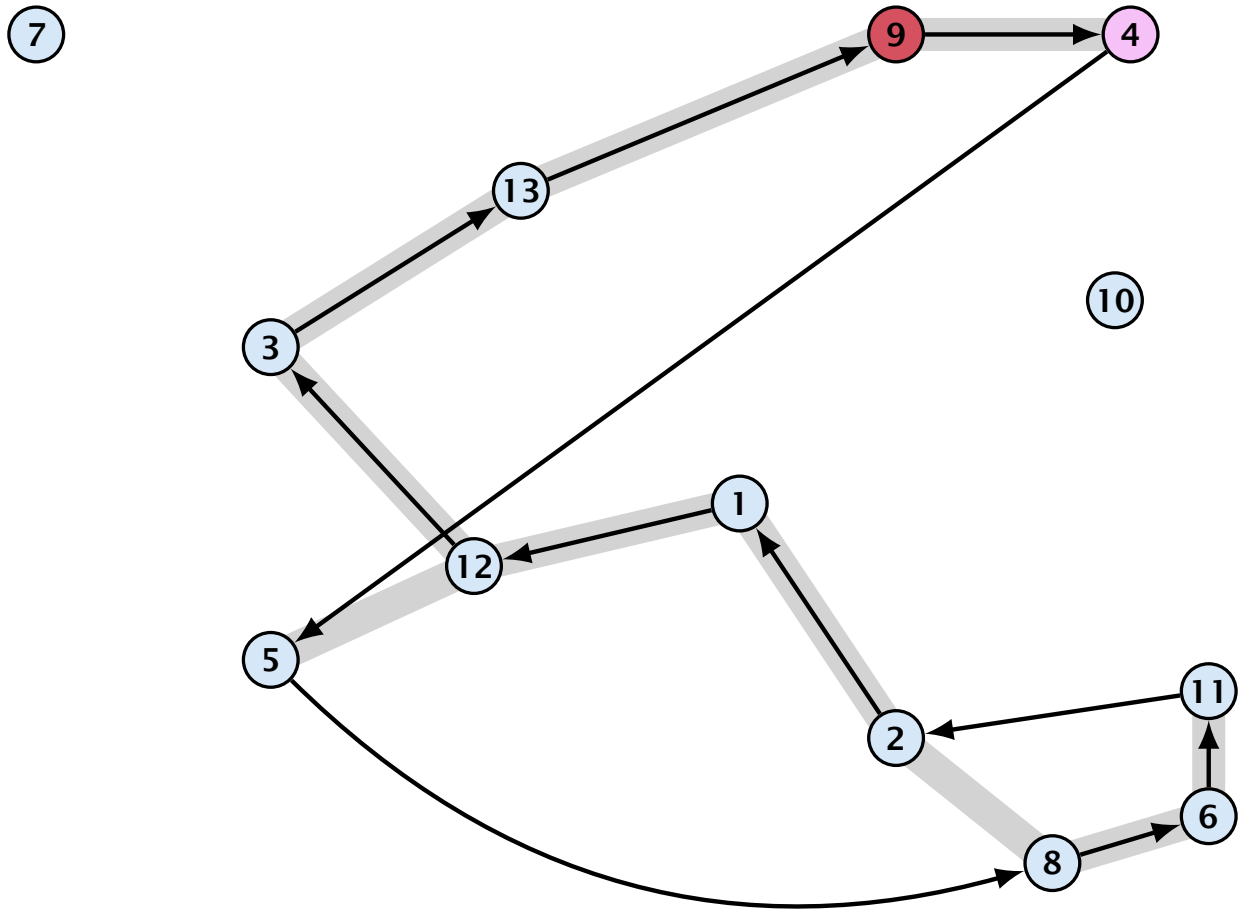

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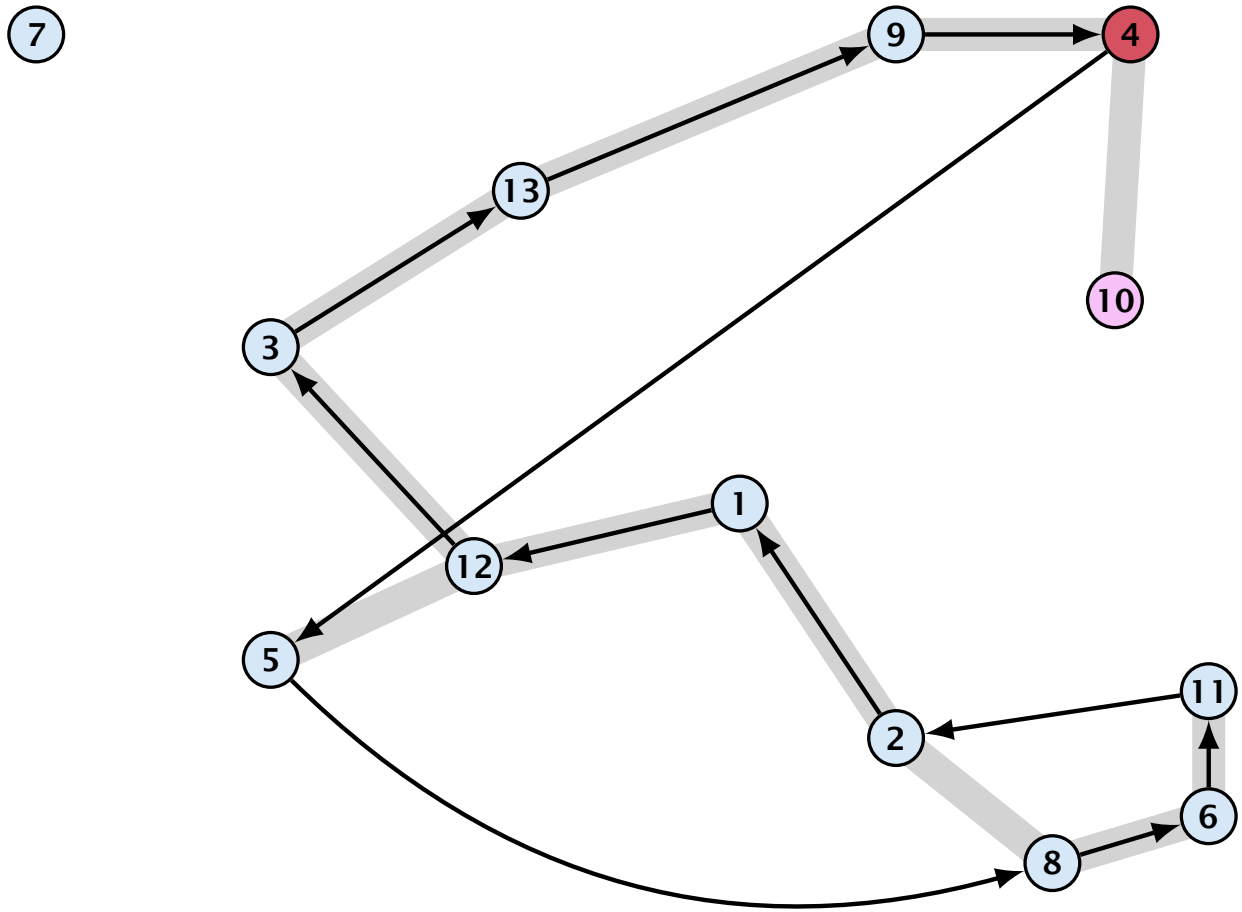

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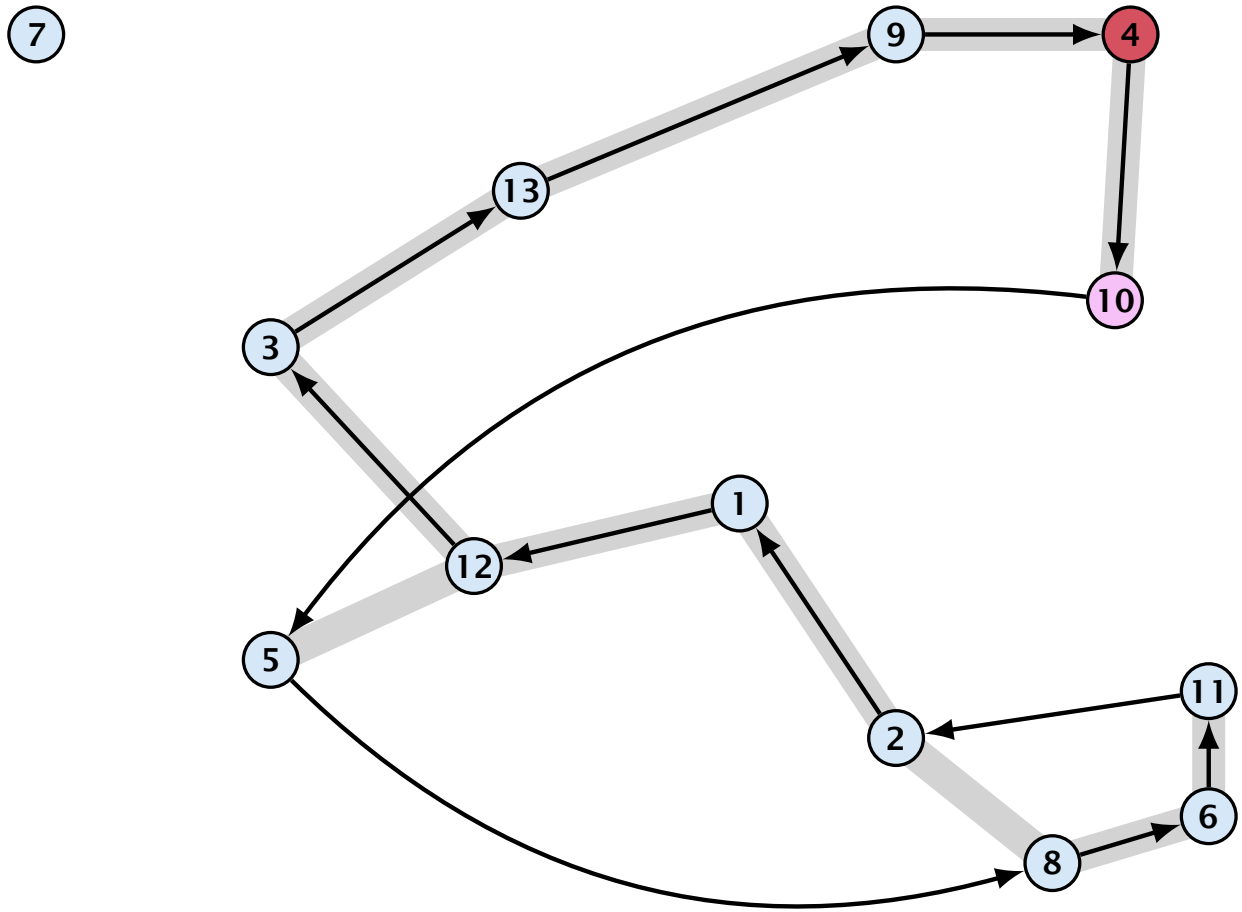

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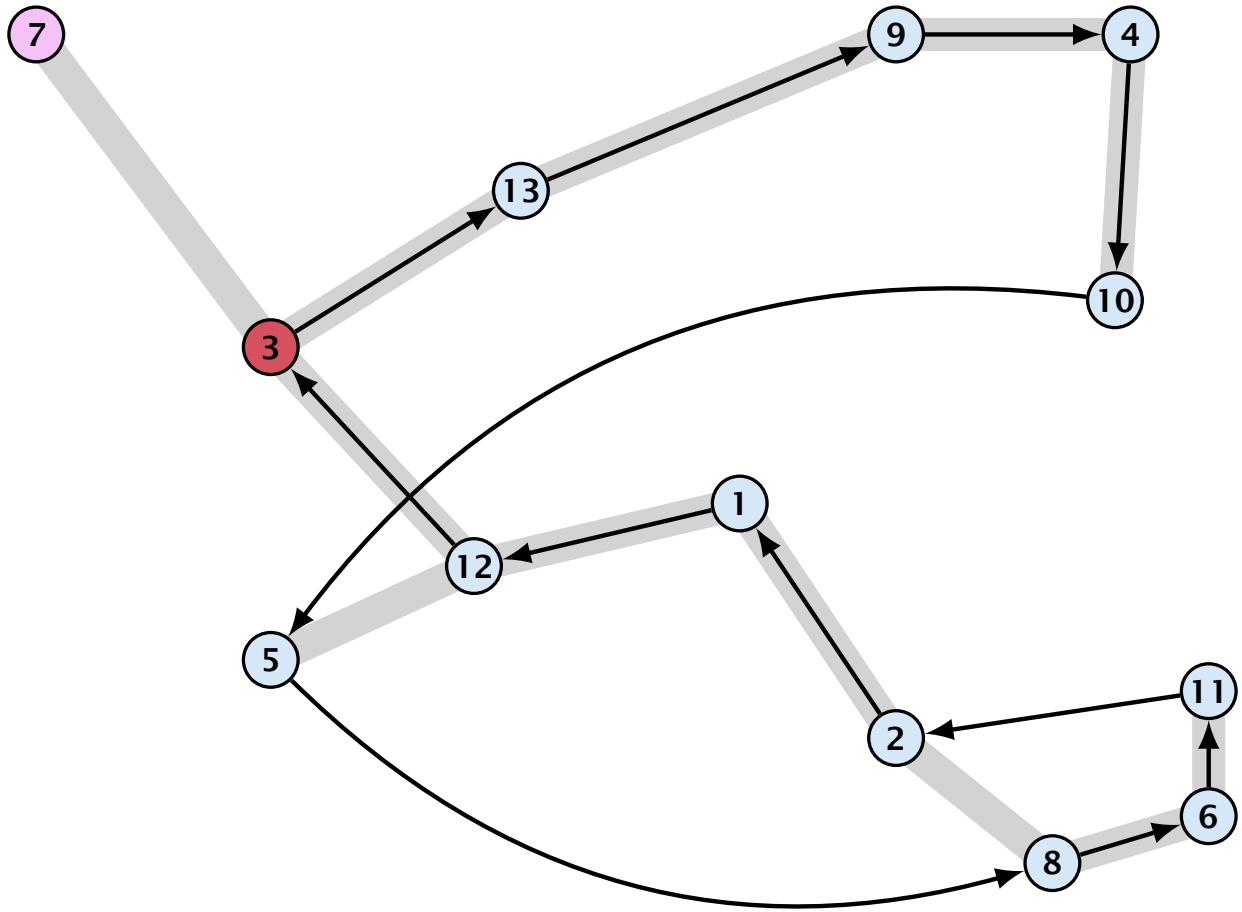

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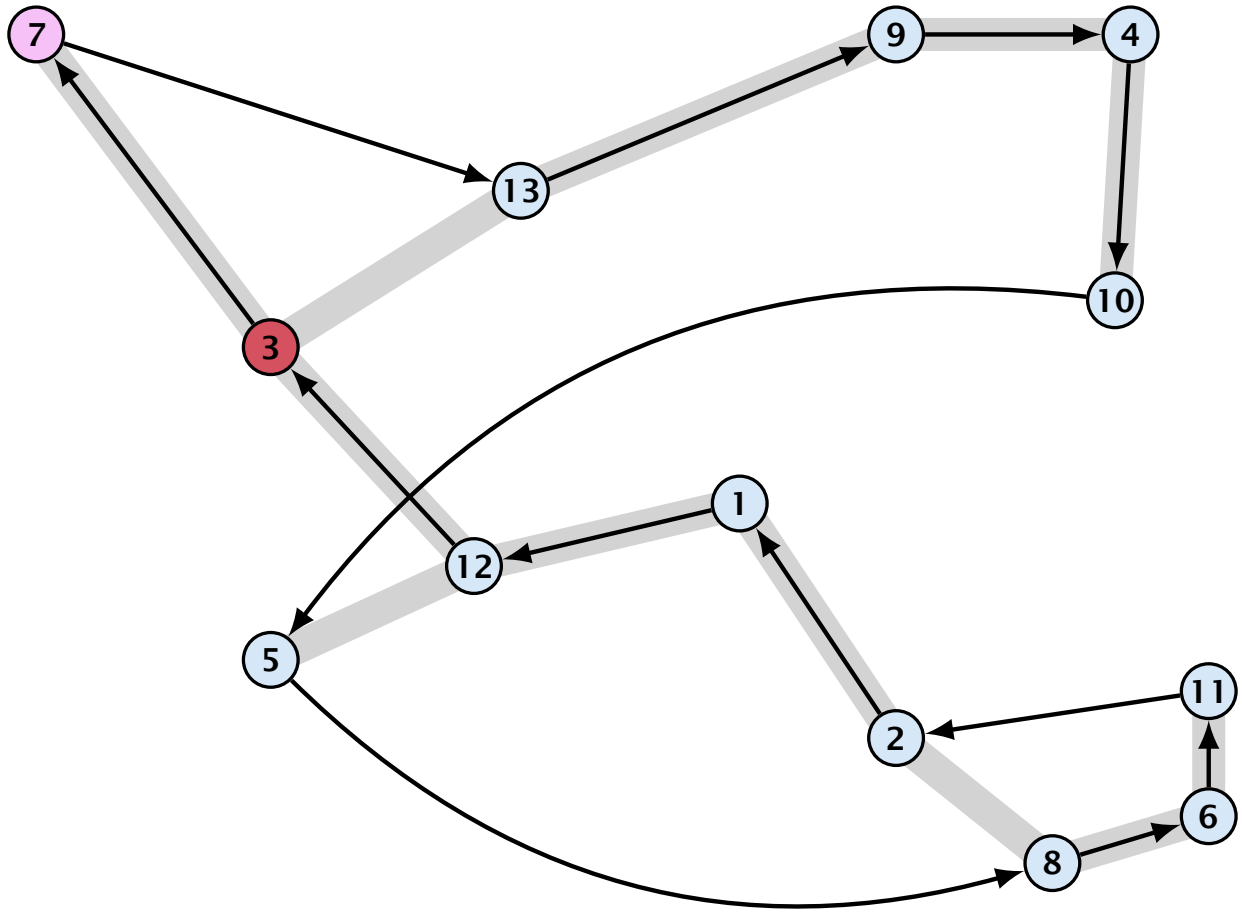

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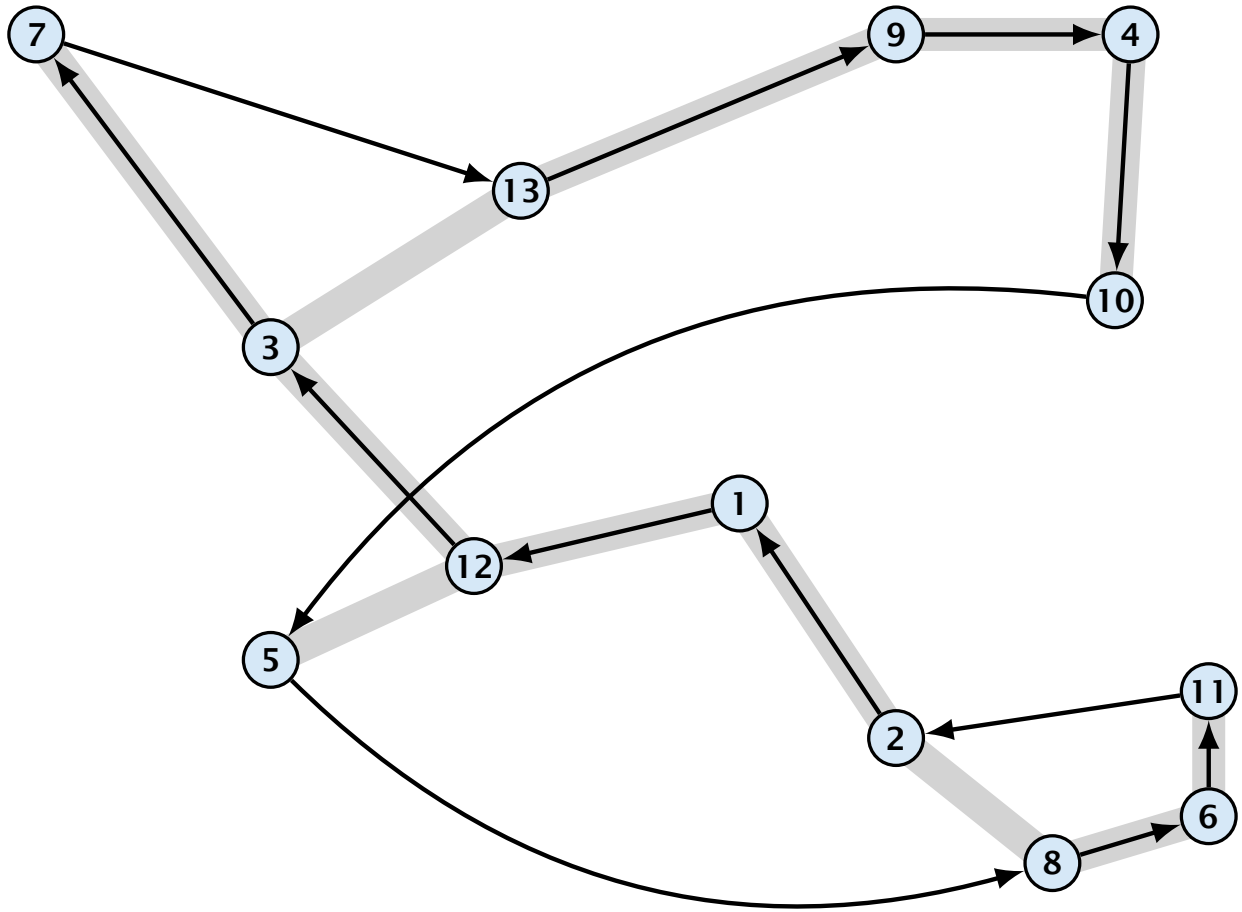

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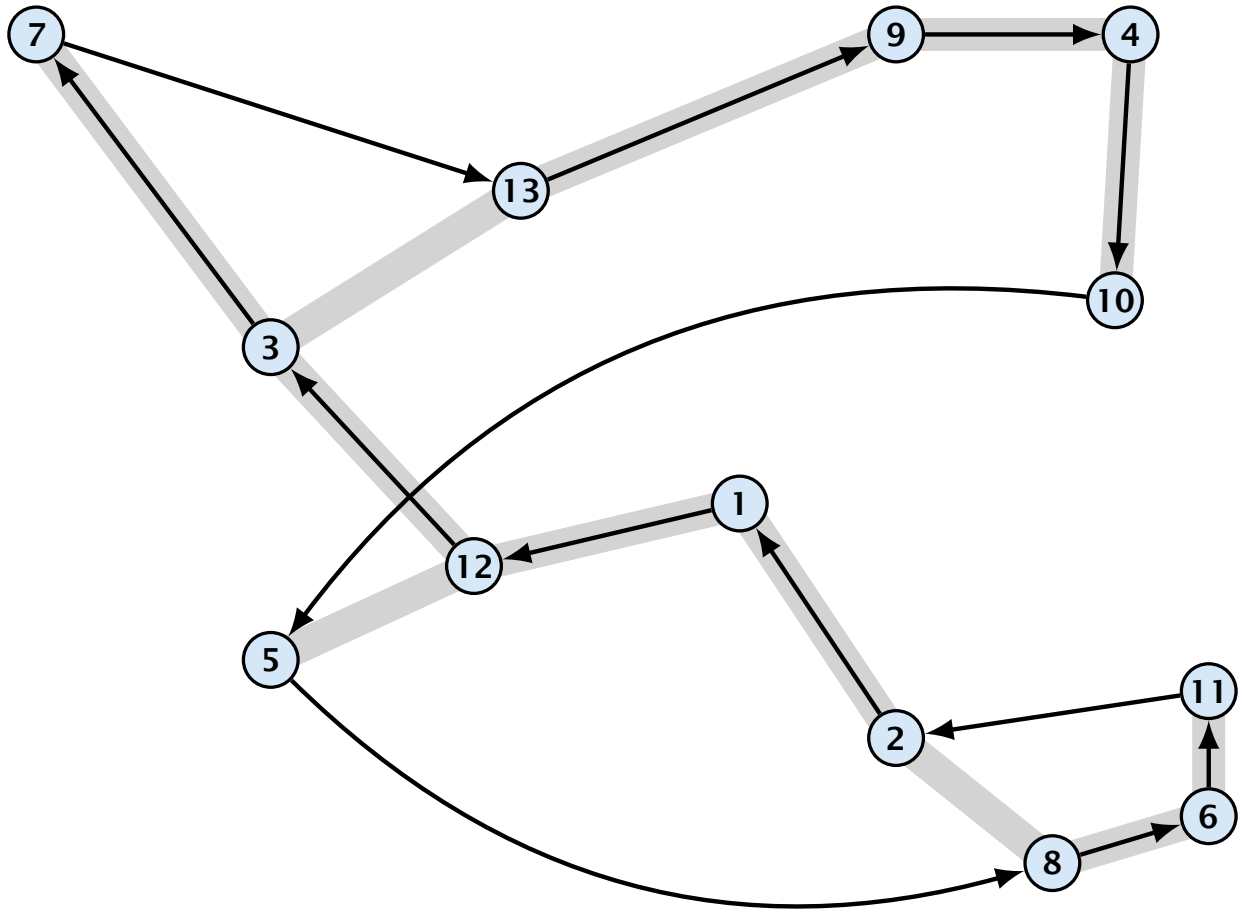

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Lemma 4

The Greedy algorithm is a 2-approximation algorithm.

Let S_i be the set at the start of the i-th iteration, and let v_i denote the node added during the iteration.

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This increases the cost by

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The edges (s_i, v_i) considered during the Greedy algorithm are exactly the edges considered during PRIMs MST algorithm.

Hence,

$$\sum_i c_{s_i,v_i} = \mathrm{OPT}_{\mathrm{MST}}(G)$$

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TSP: A different approach

Suppose that we are given an Eulerian graph $G' = (V, E', c')$ of $G = (V, E, c)$ such that for any edge $(i, j) \in E'$ $c'(i, j) \geq c(i, j)$.

Then we can find a TSP-tour of cost at most

$$\sum_{e \in E'} c'(e)$$

- Find an Euler tour of G'.
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- The cost of this TSP tour is at most the cost of the Euler tour because of triangle inequality.

This technique is known as short cutting the Euler tour.

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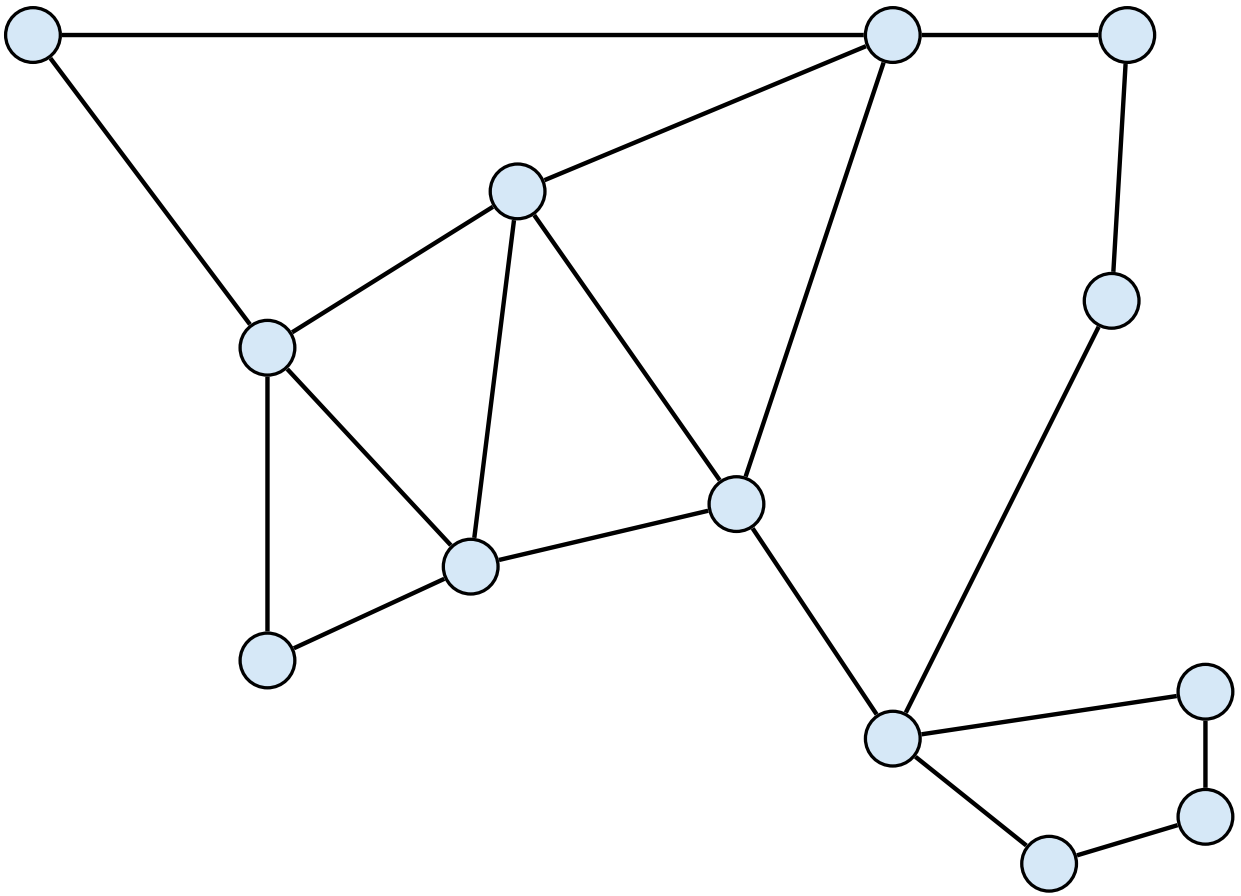

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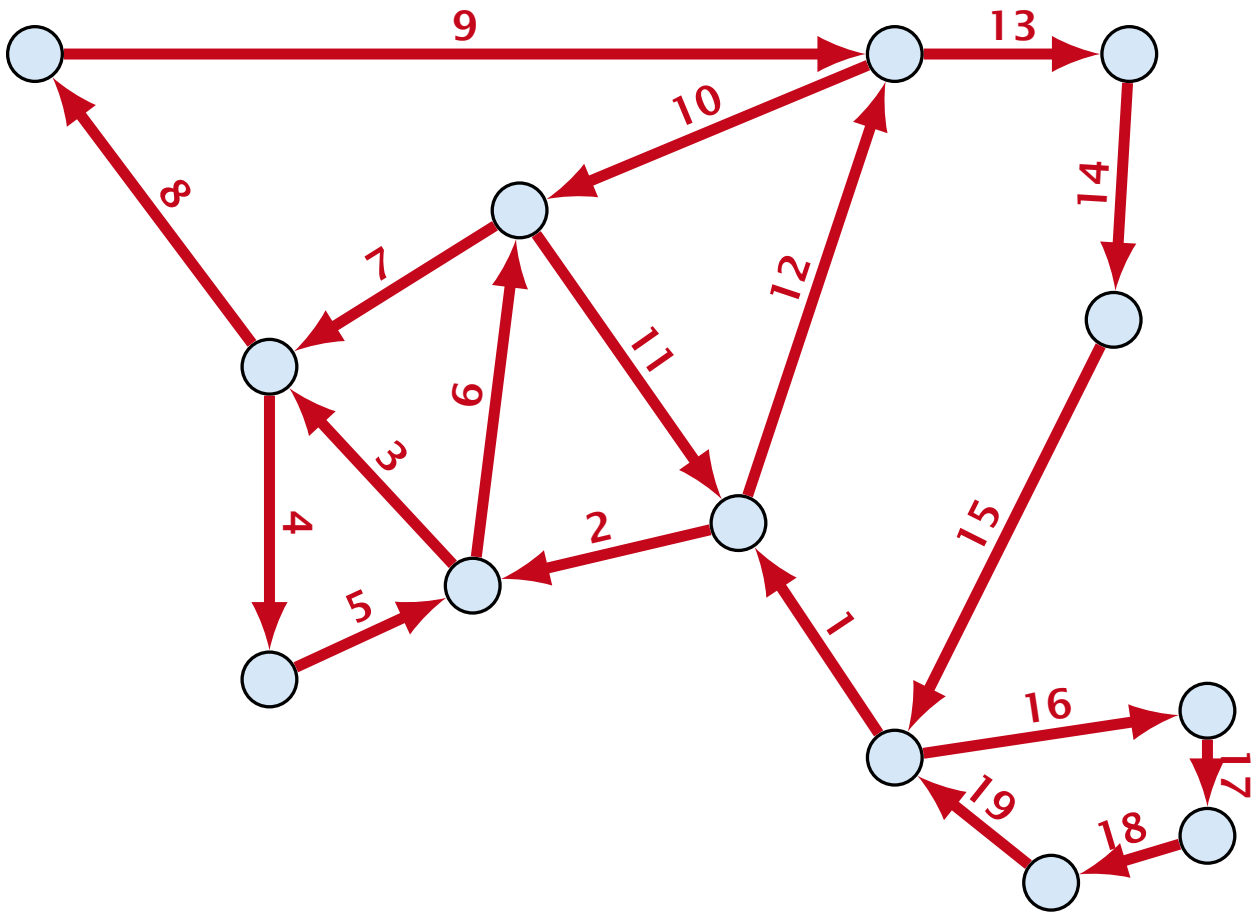

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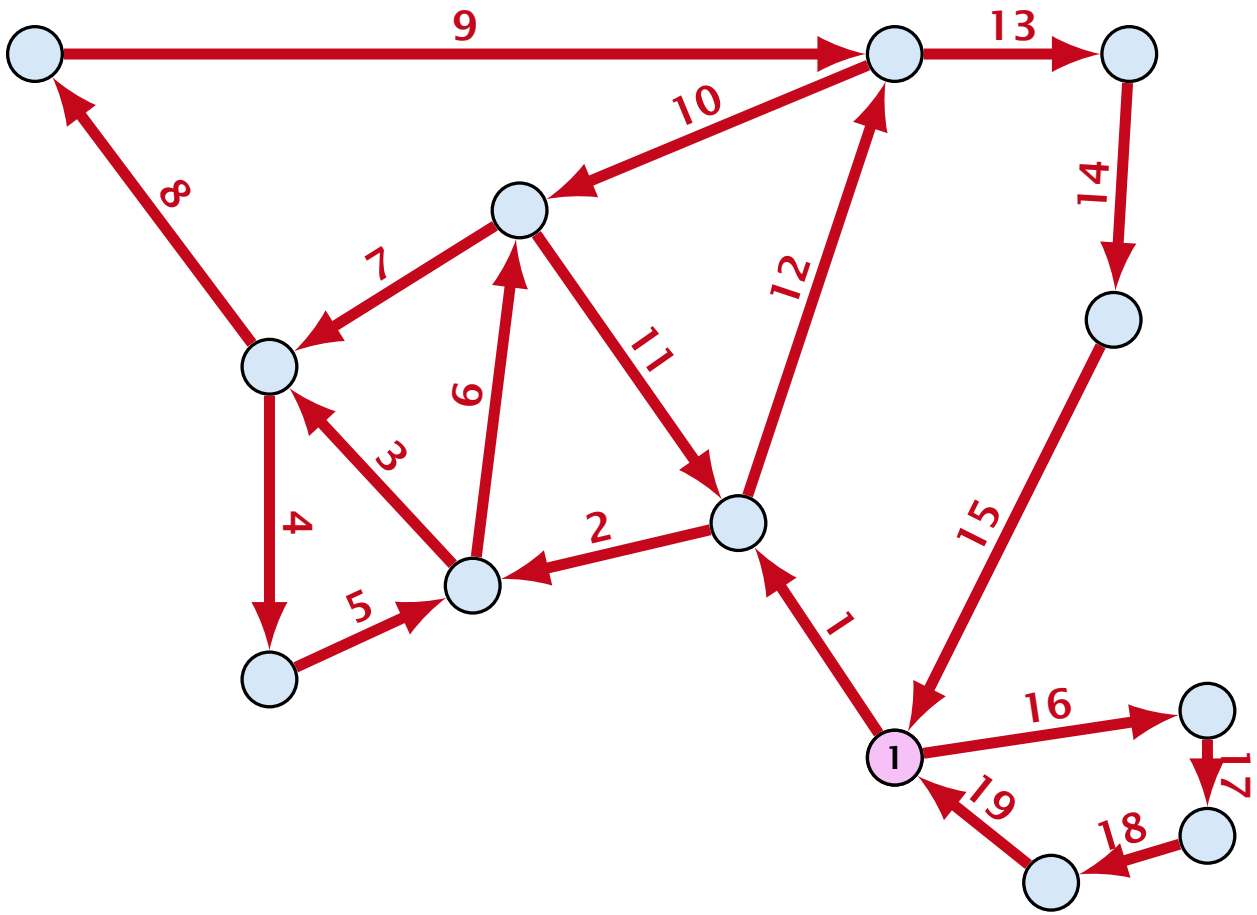

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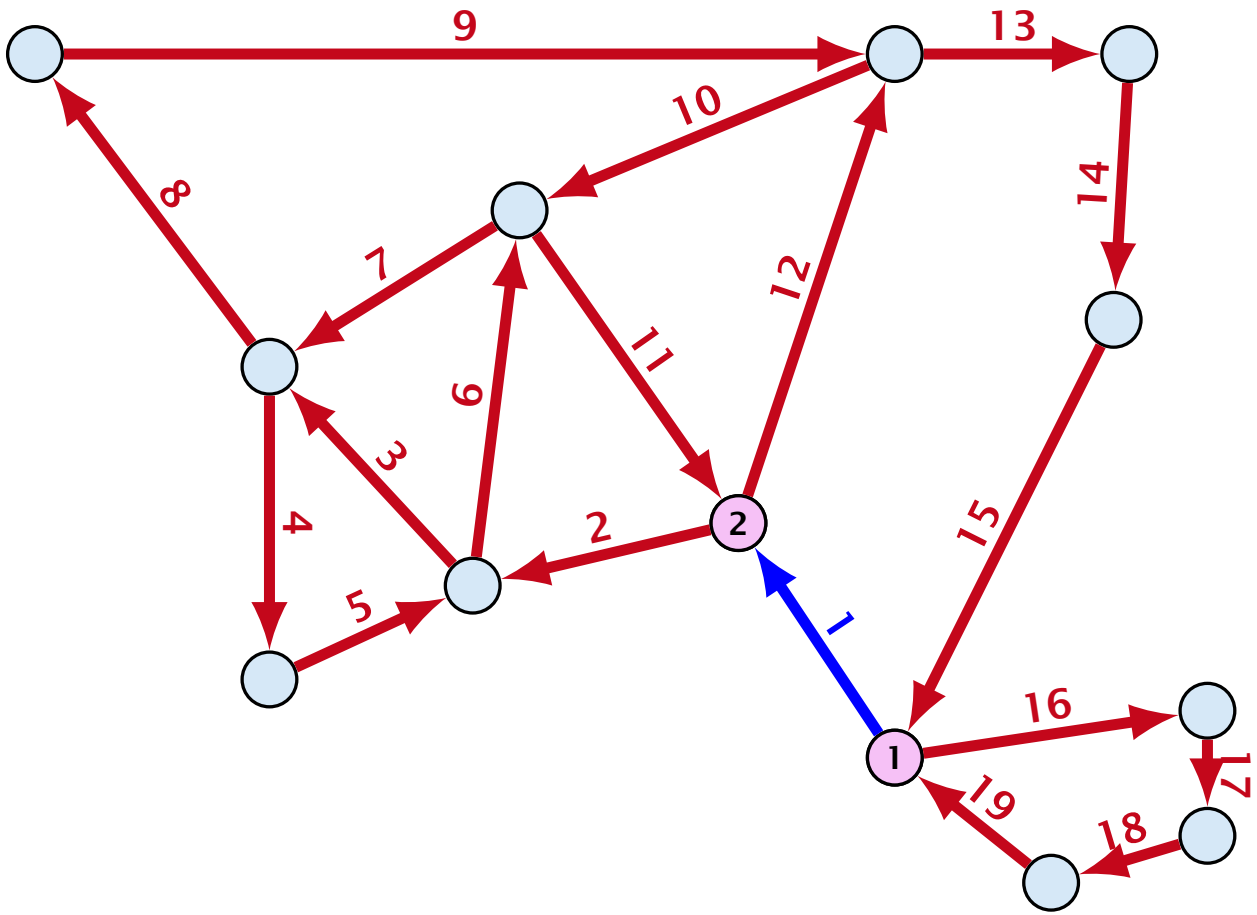

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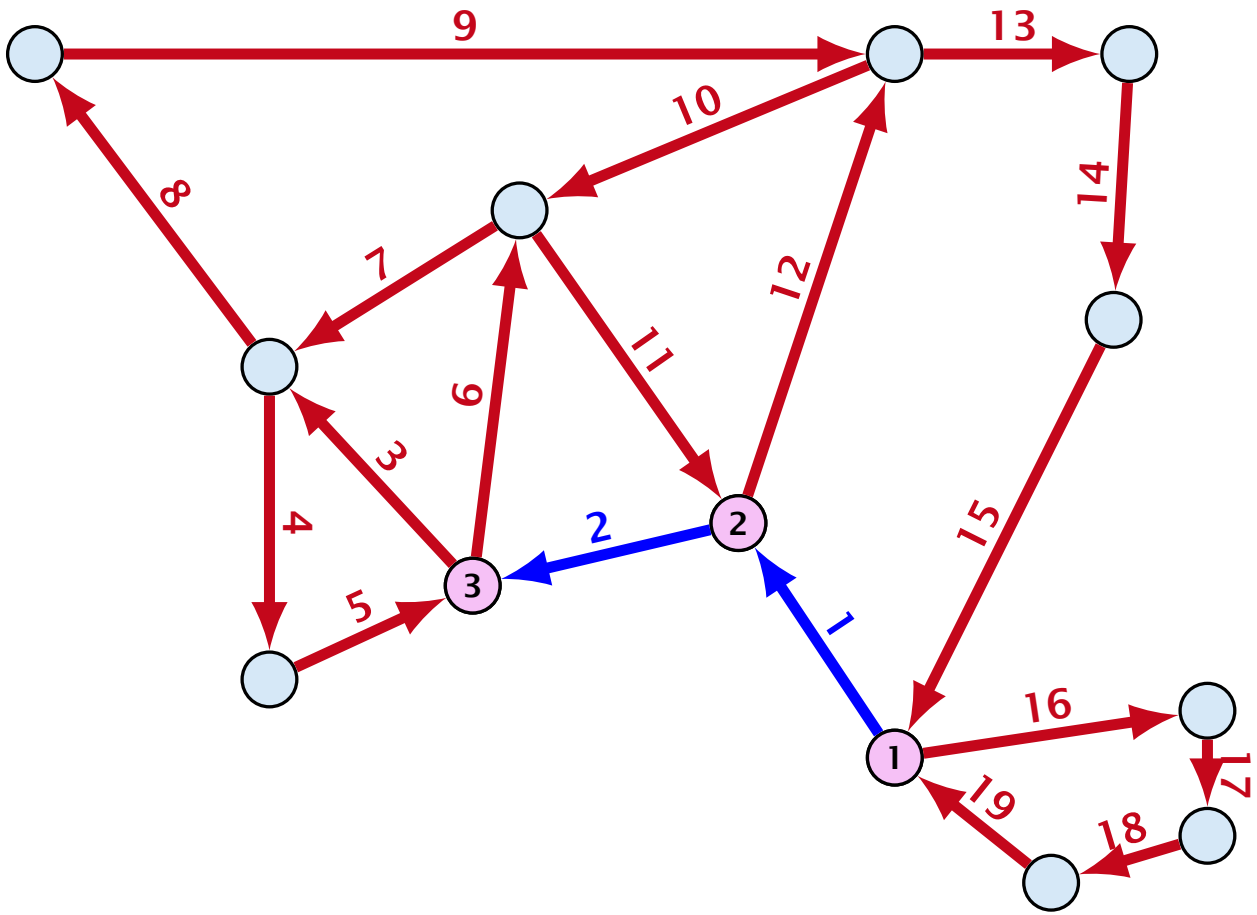

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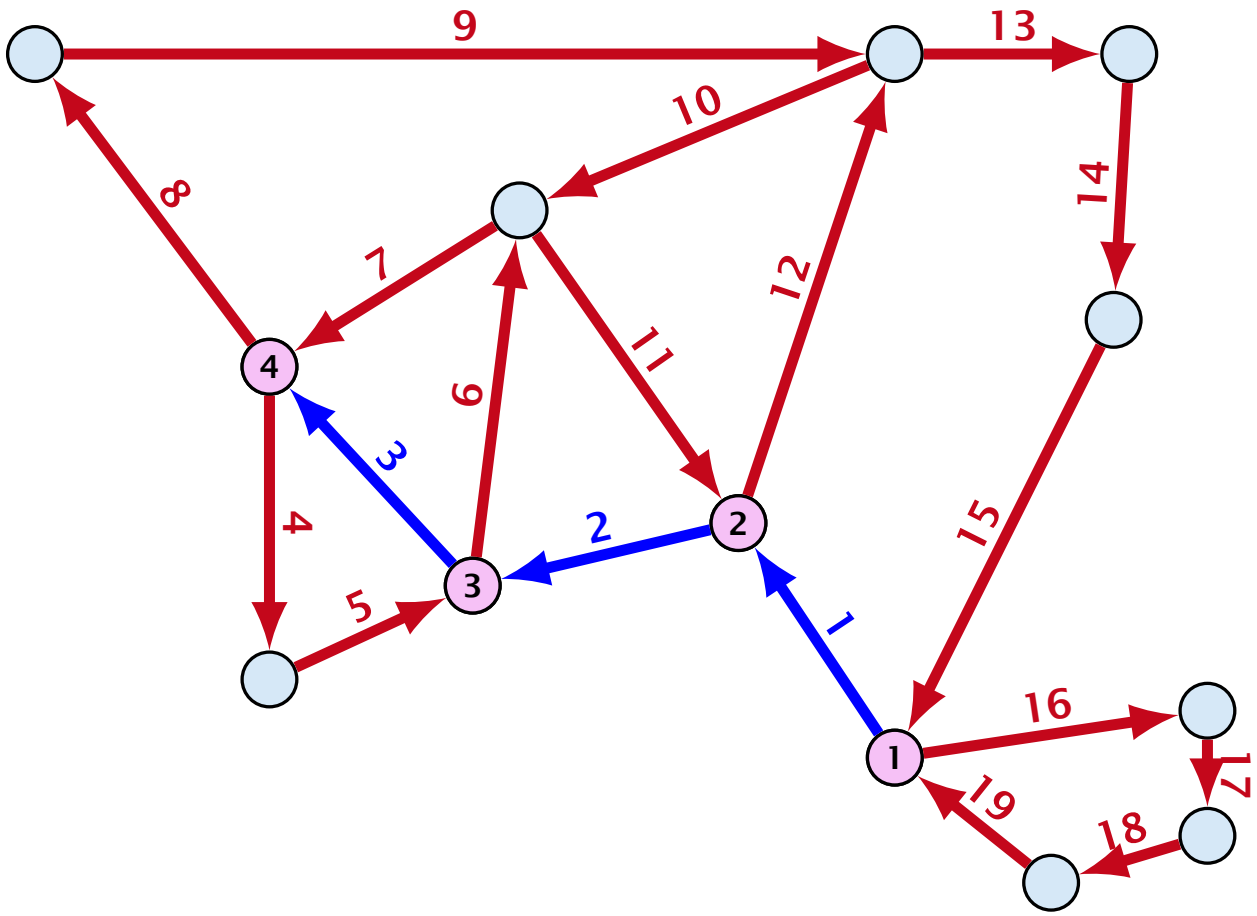

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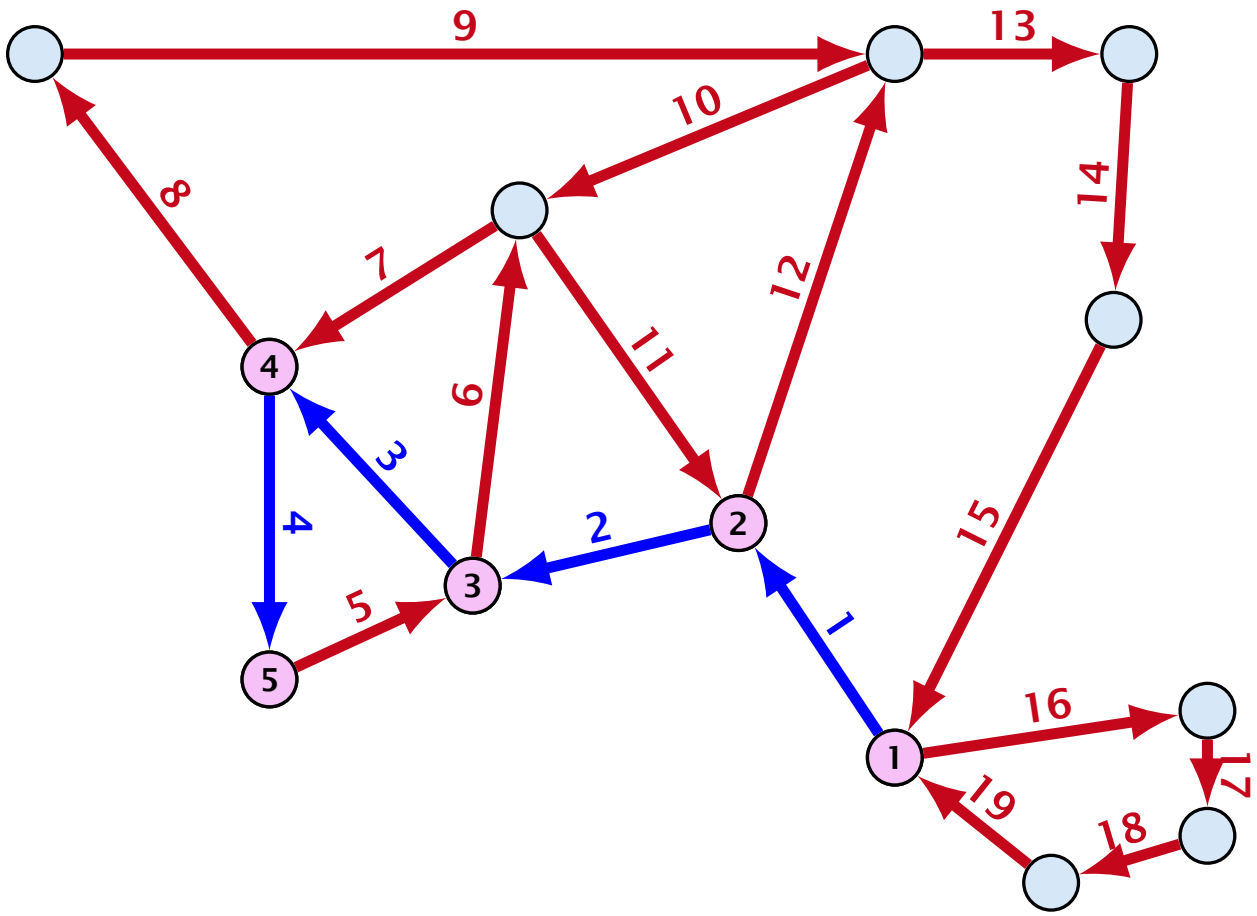

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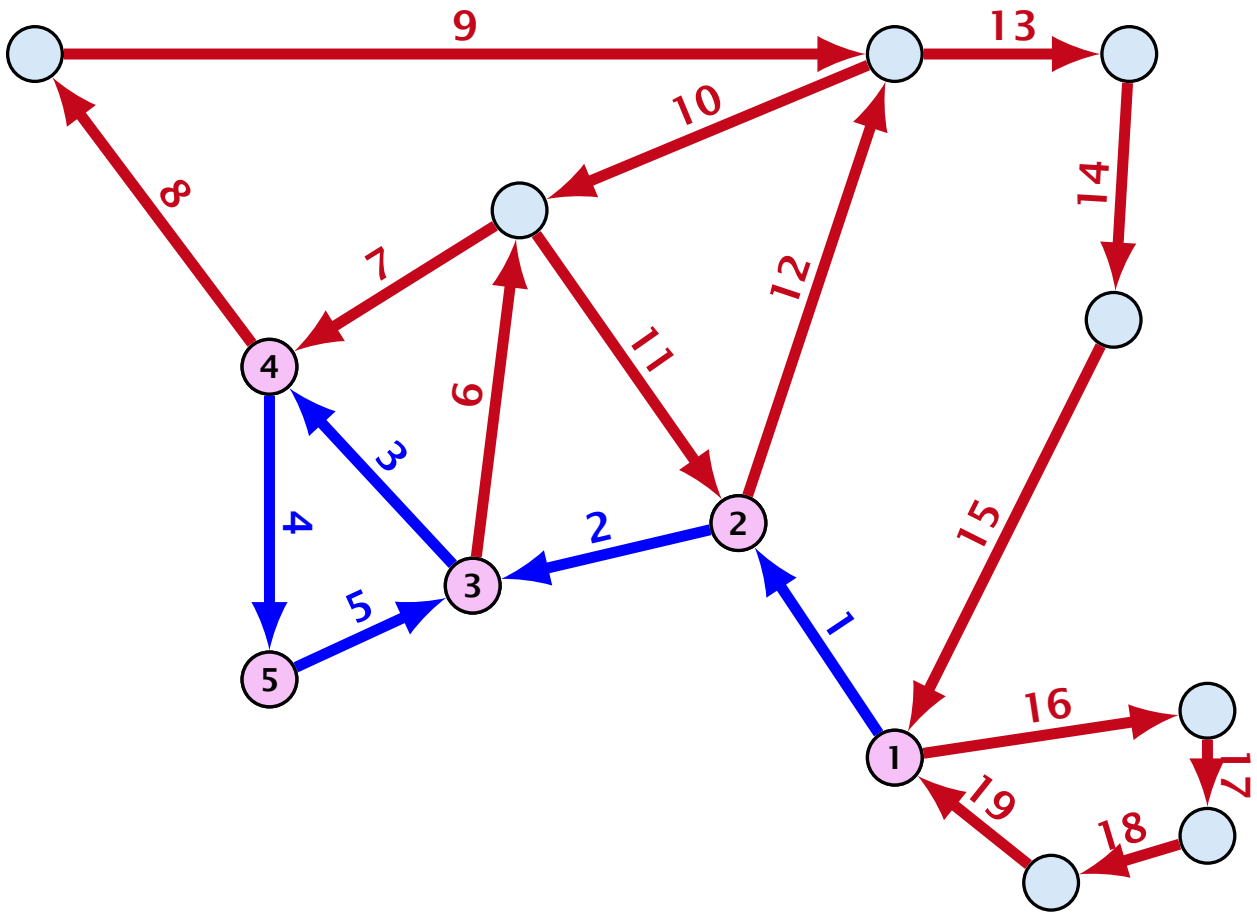

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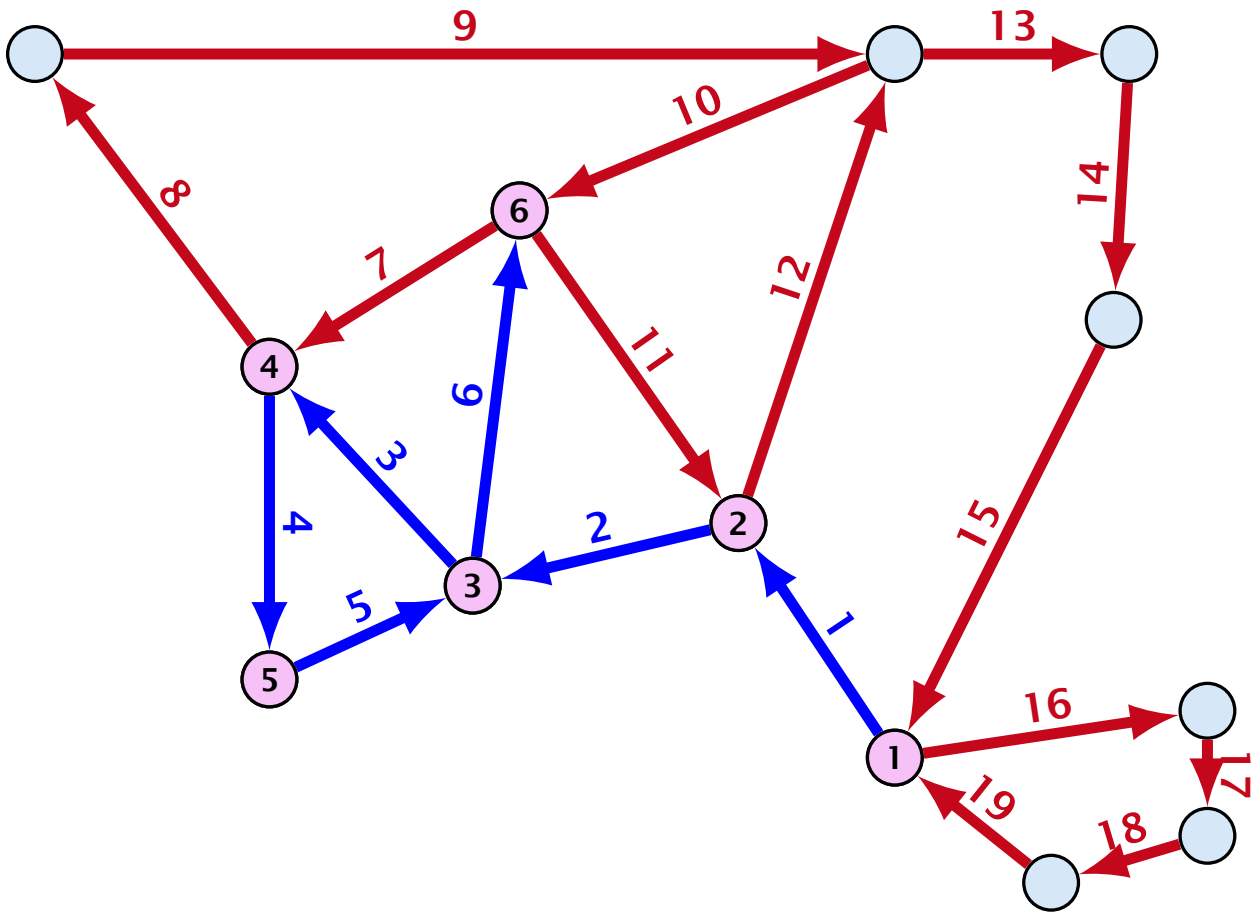

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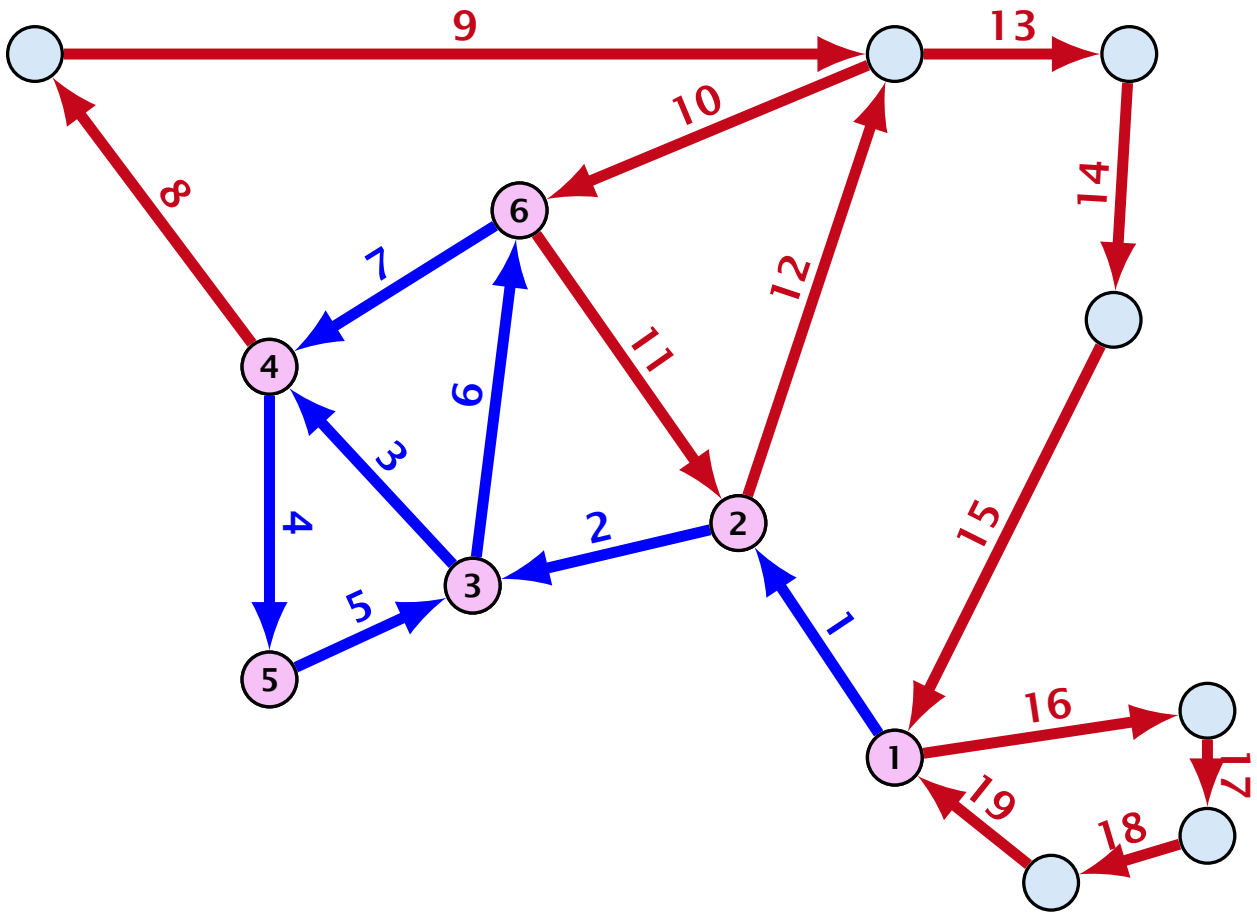

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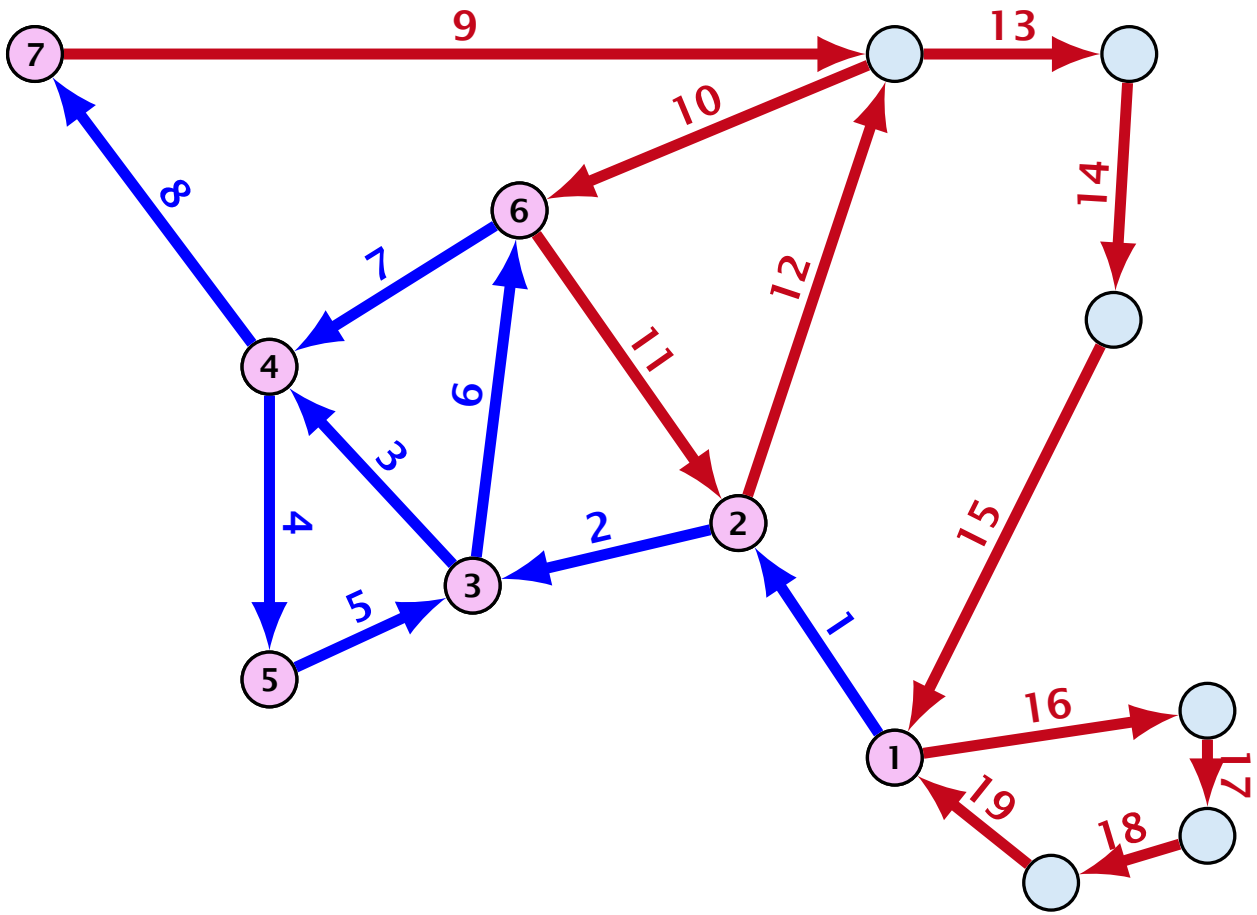

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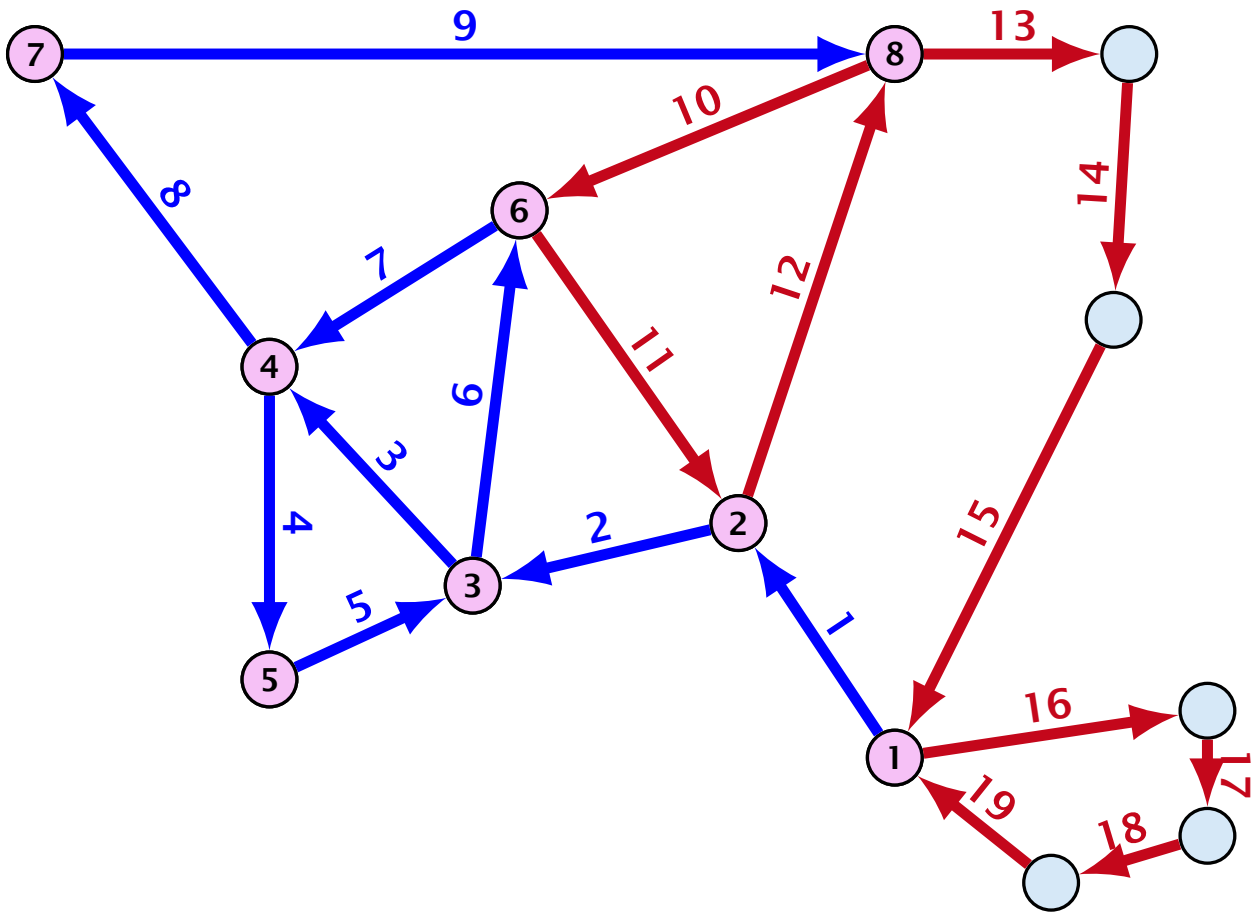

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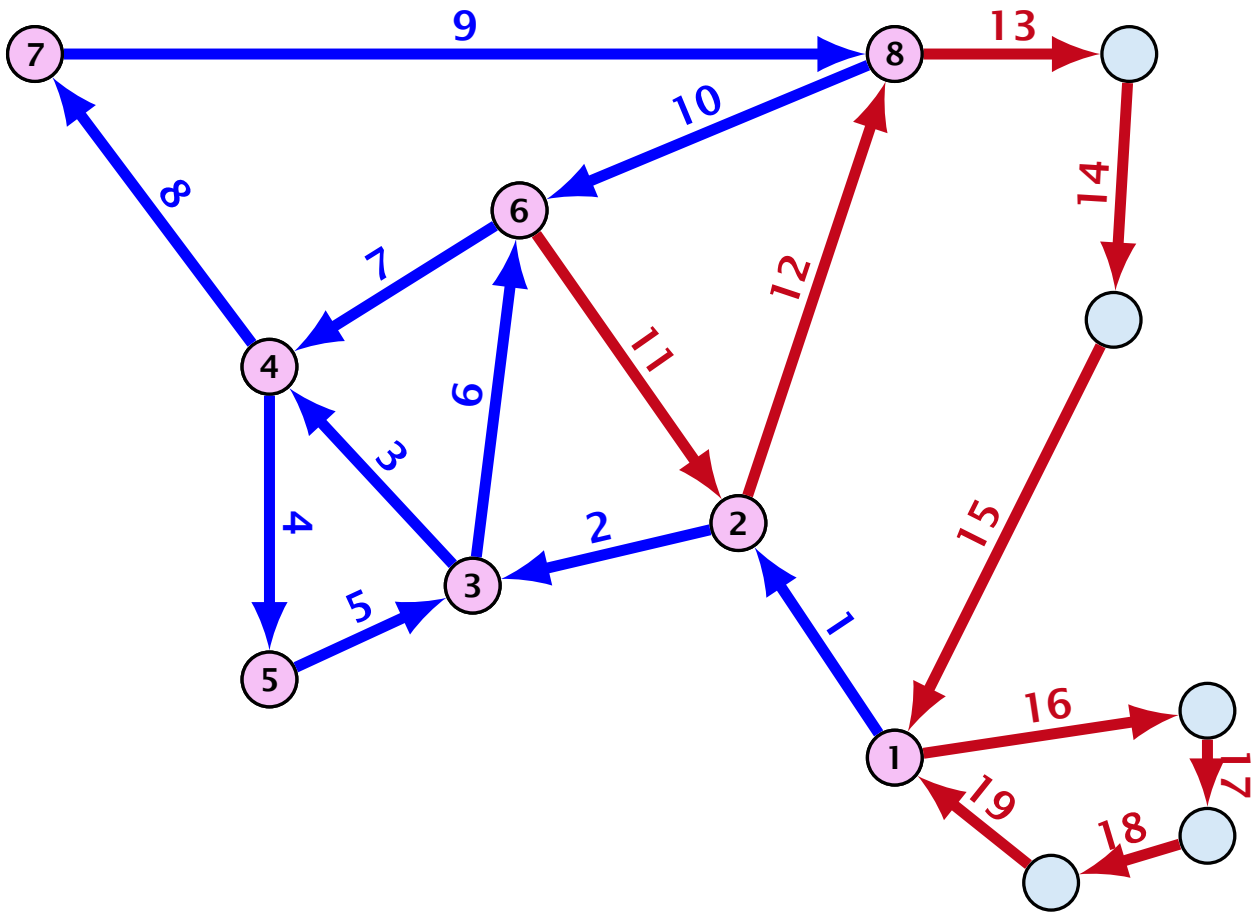

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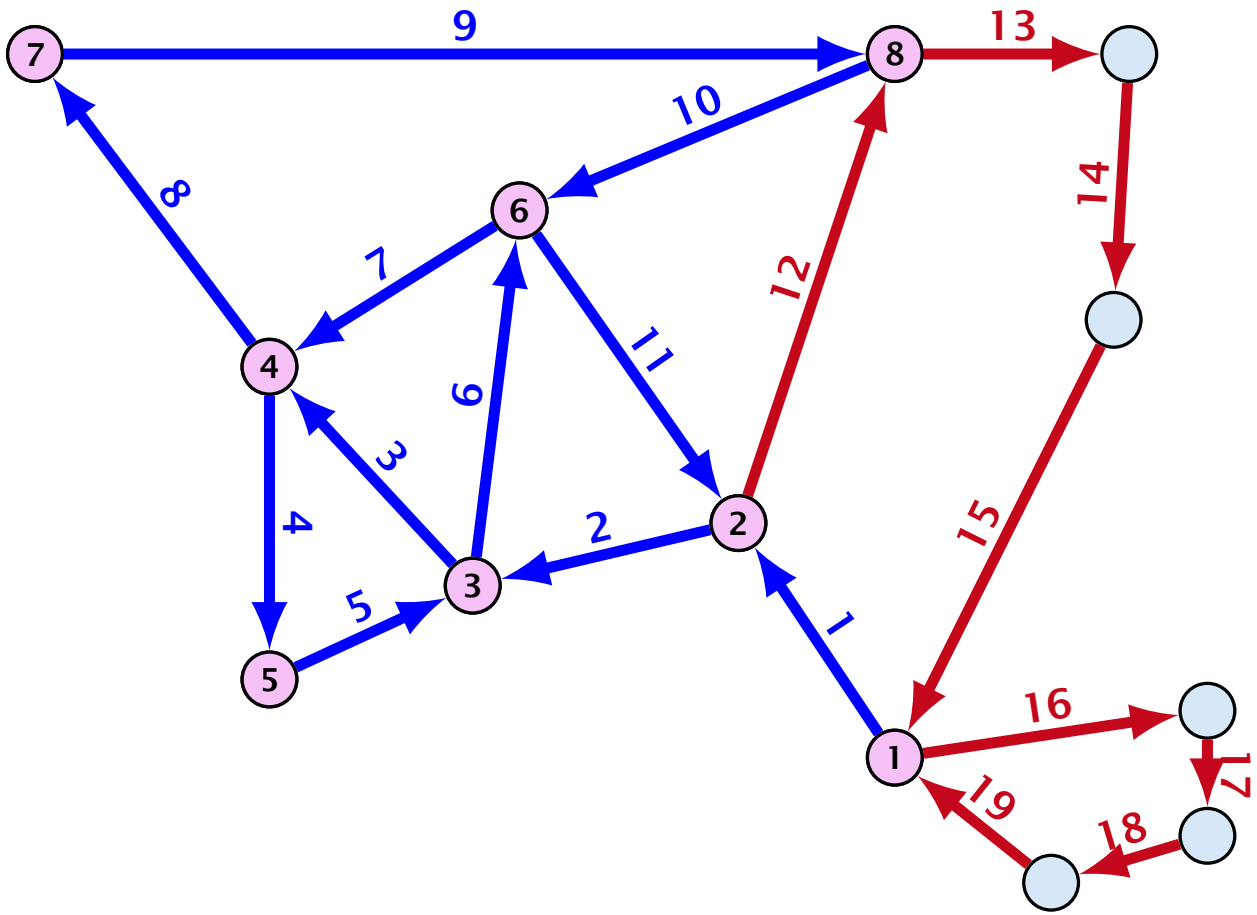

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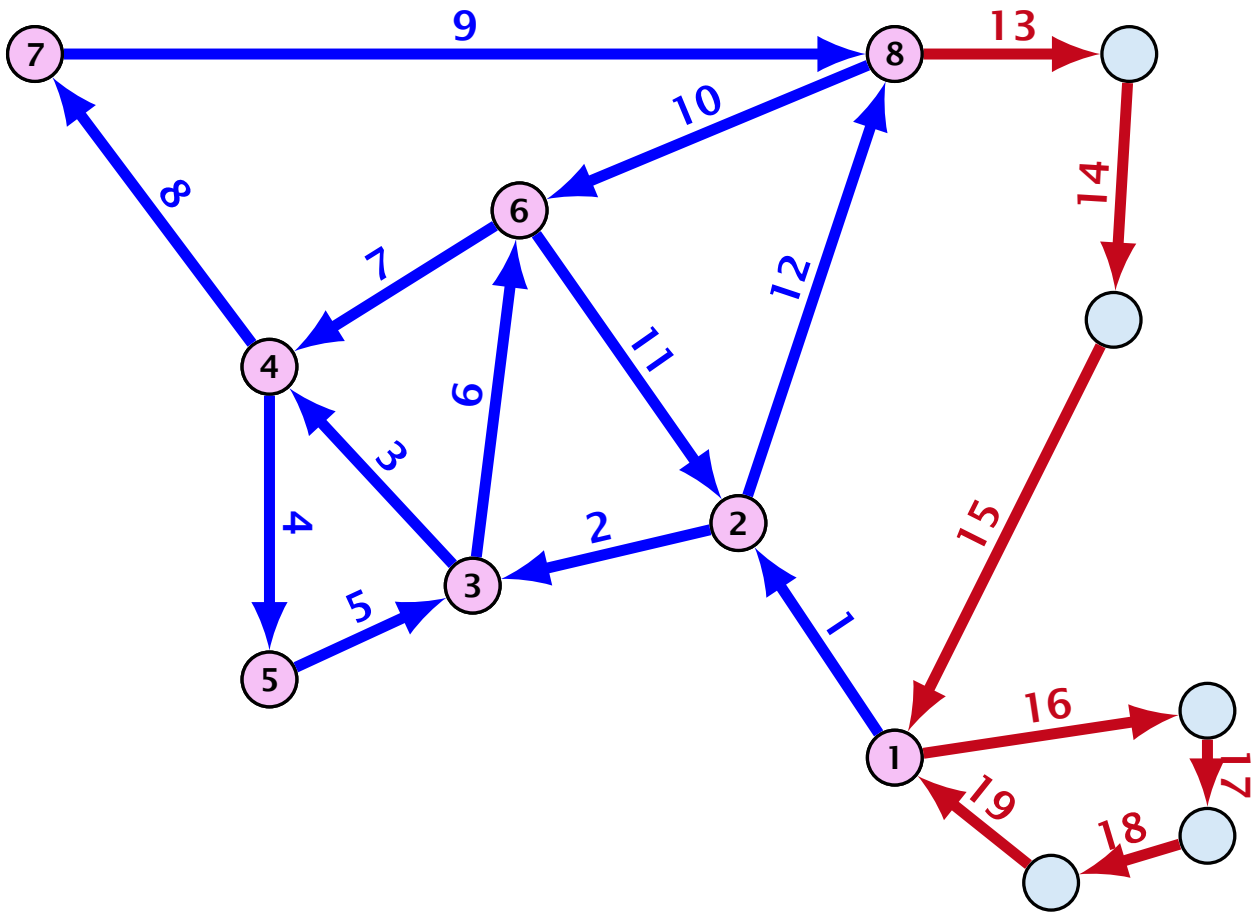

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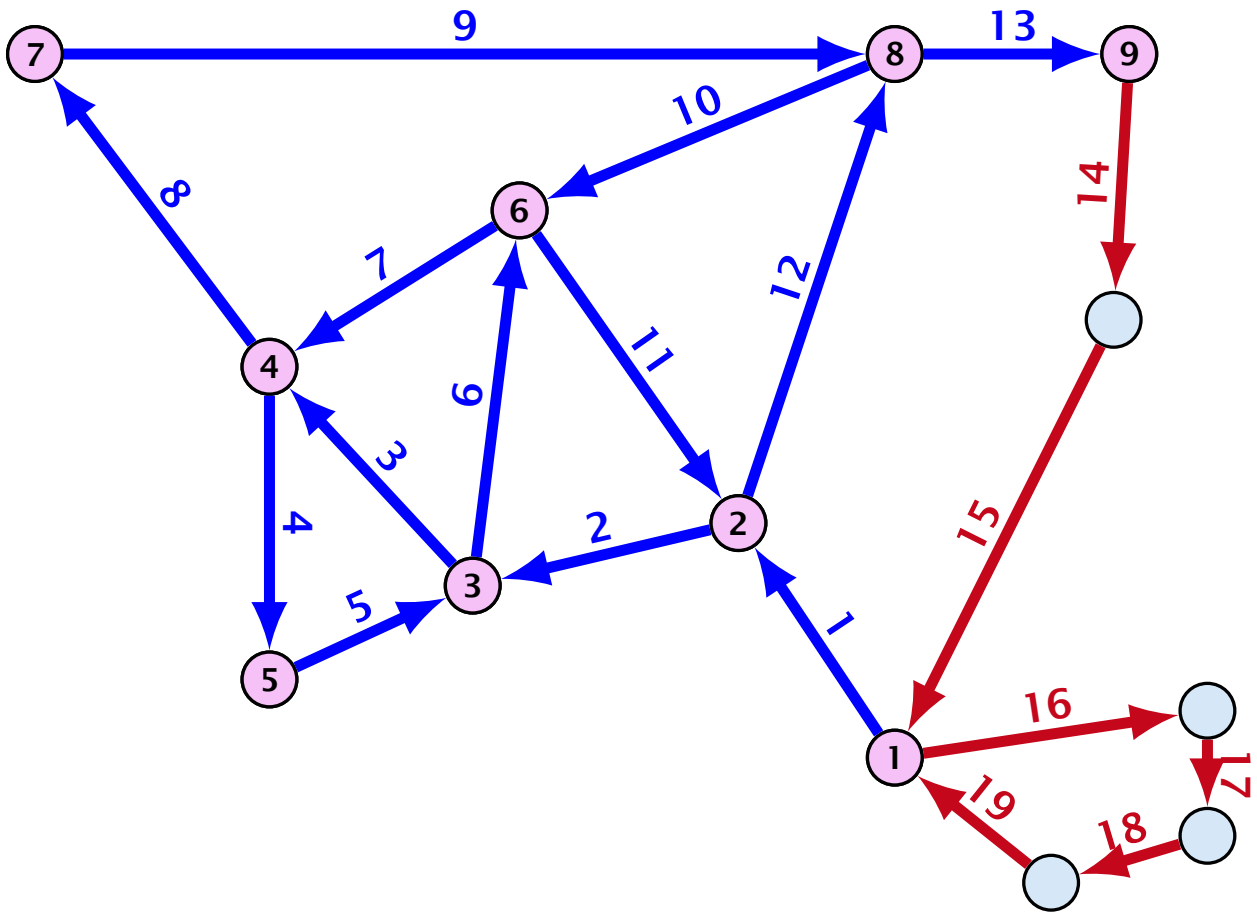

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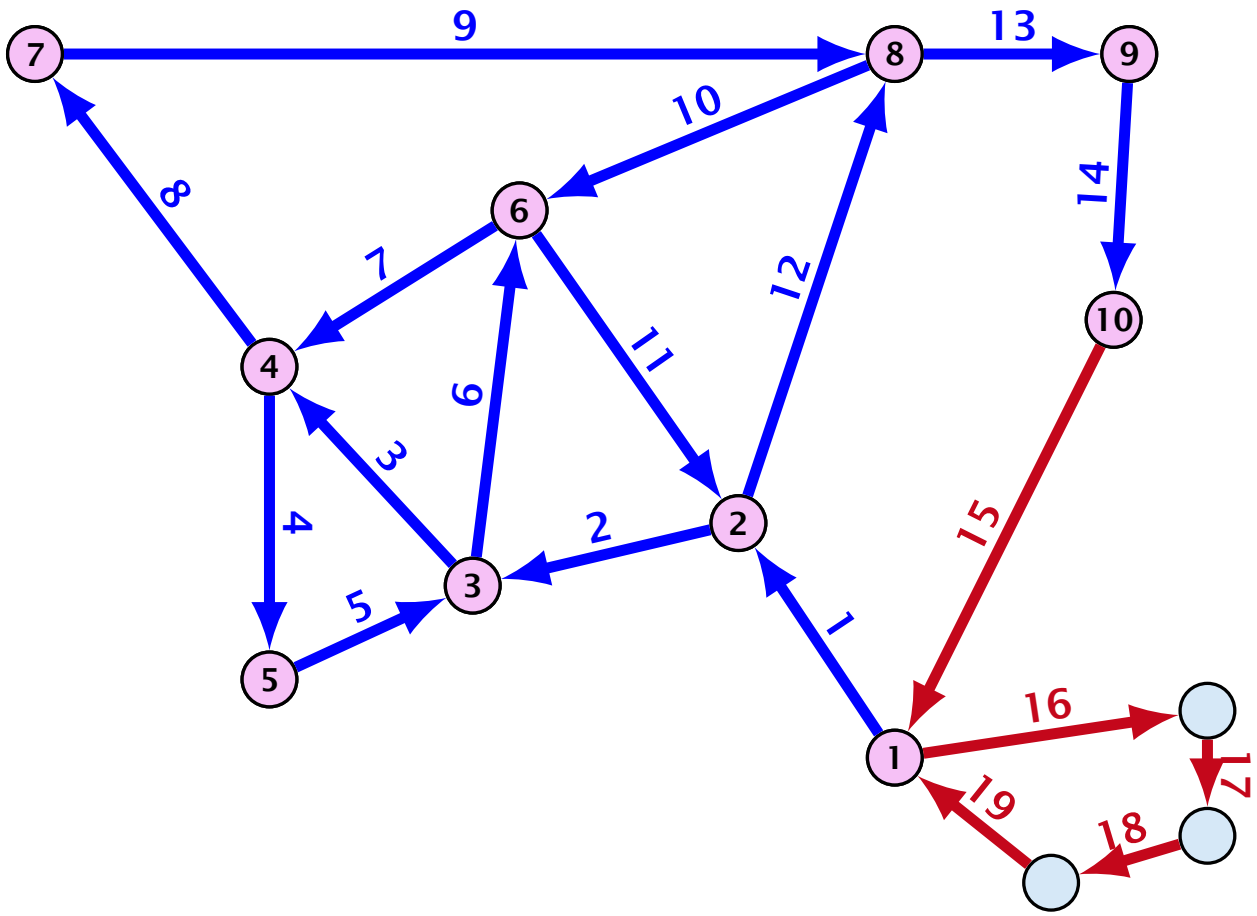

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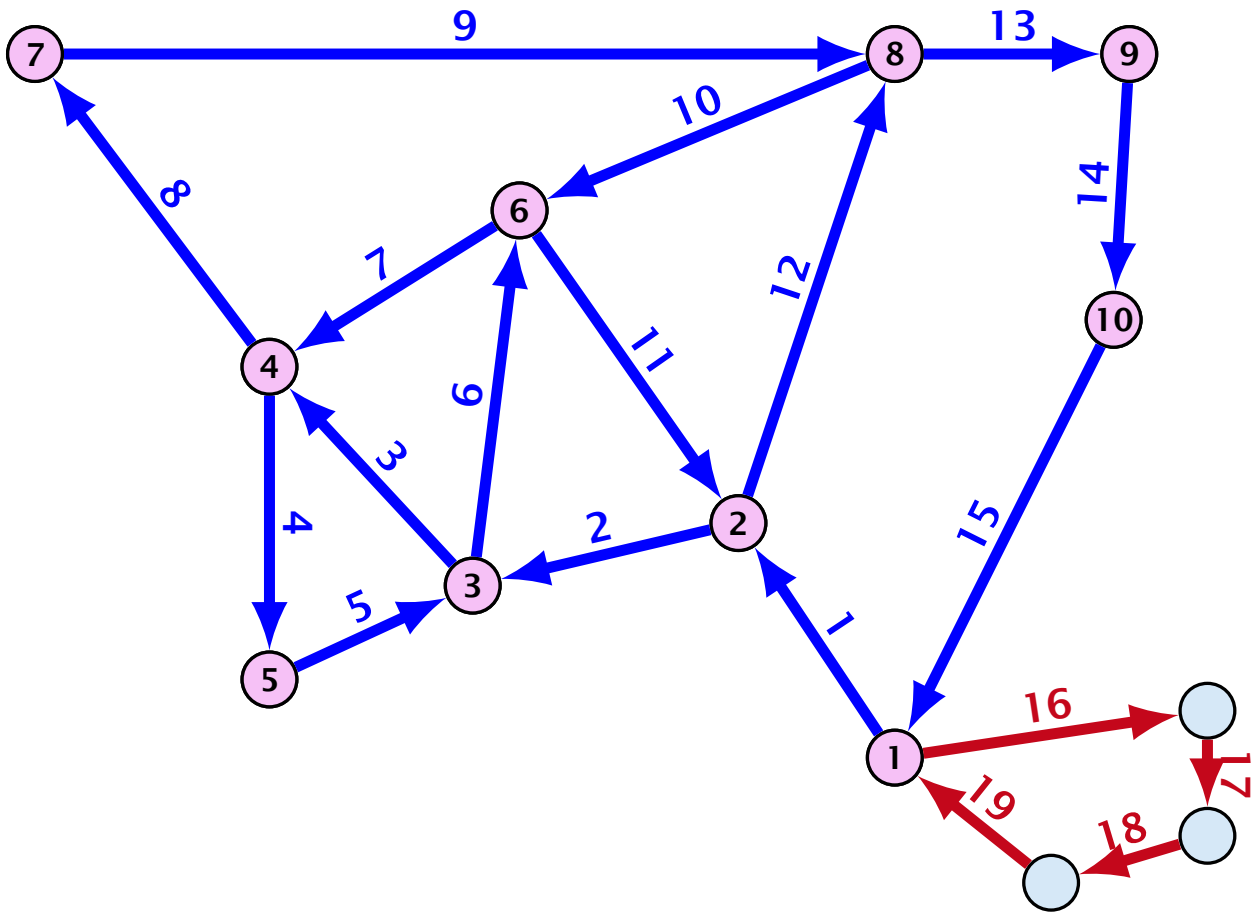

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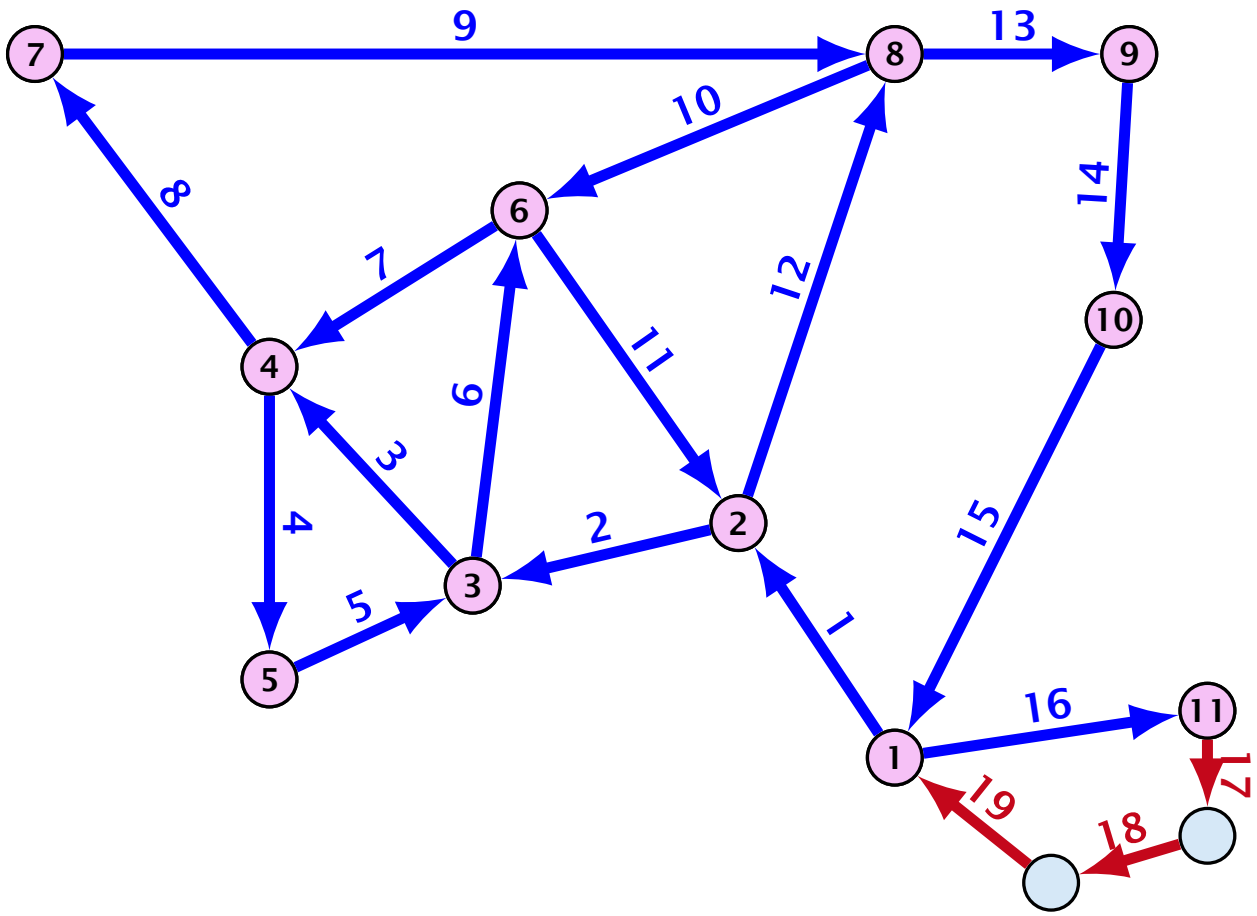

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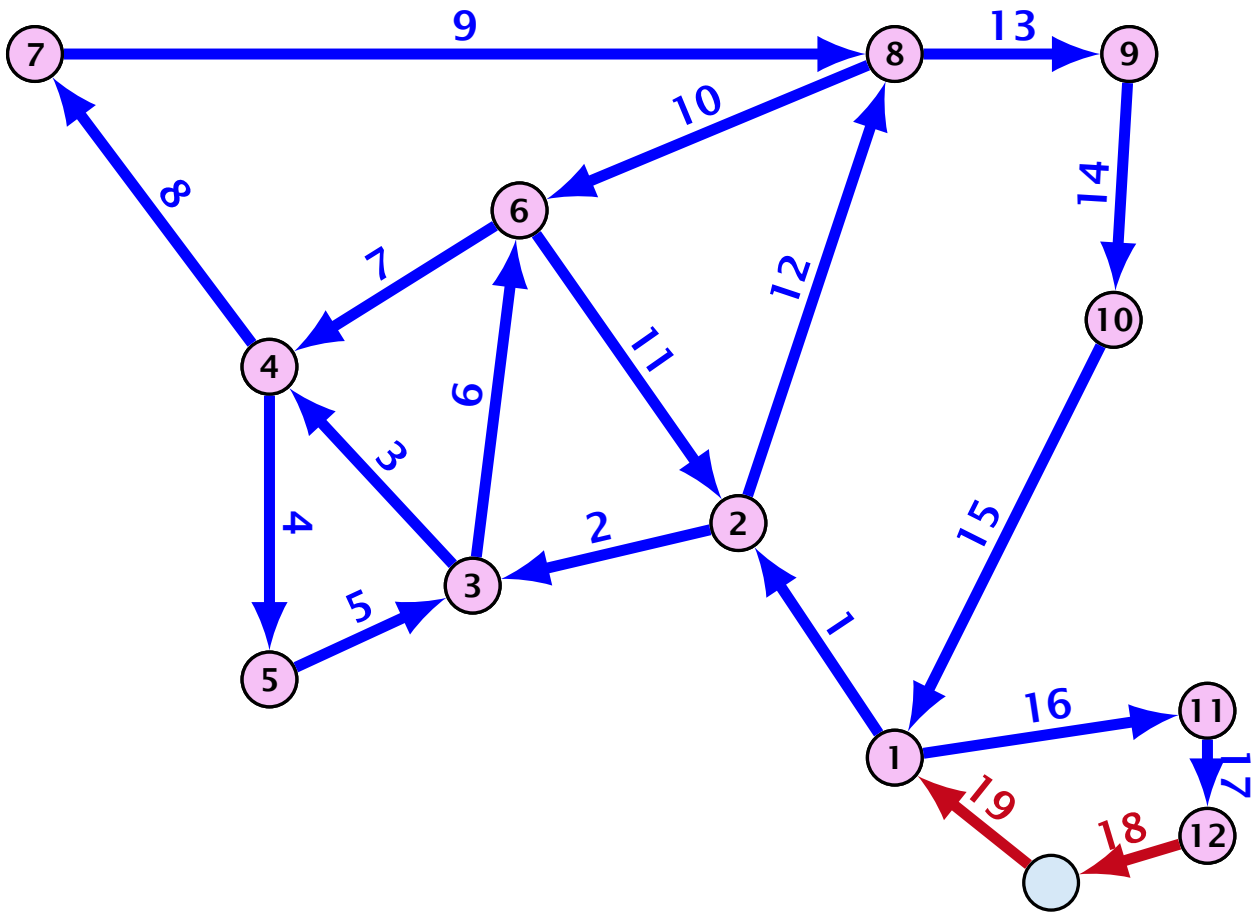

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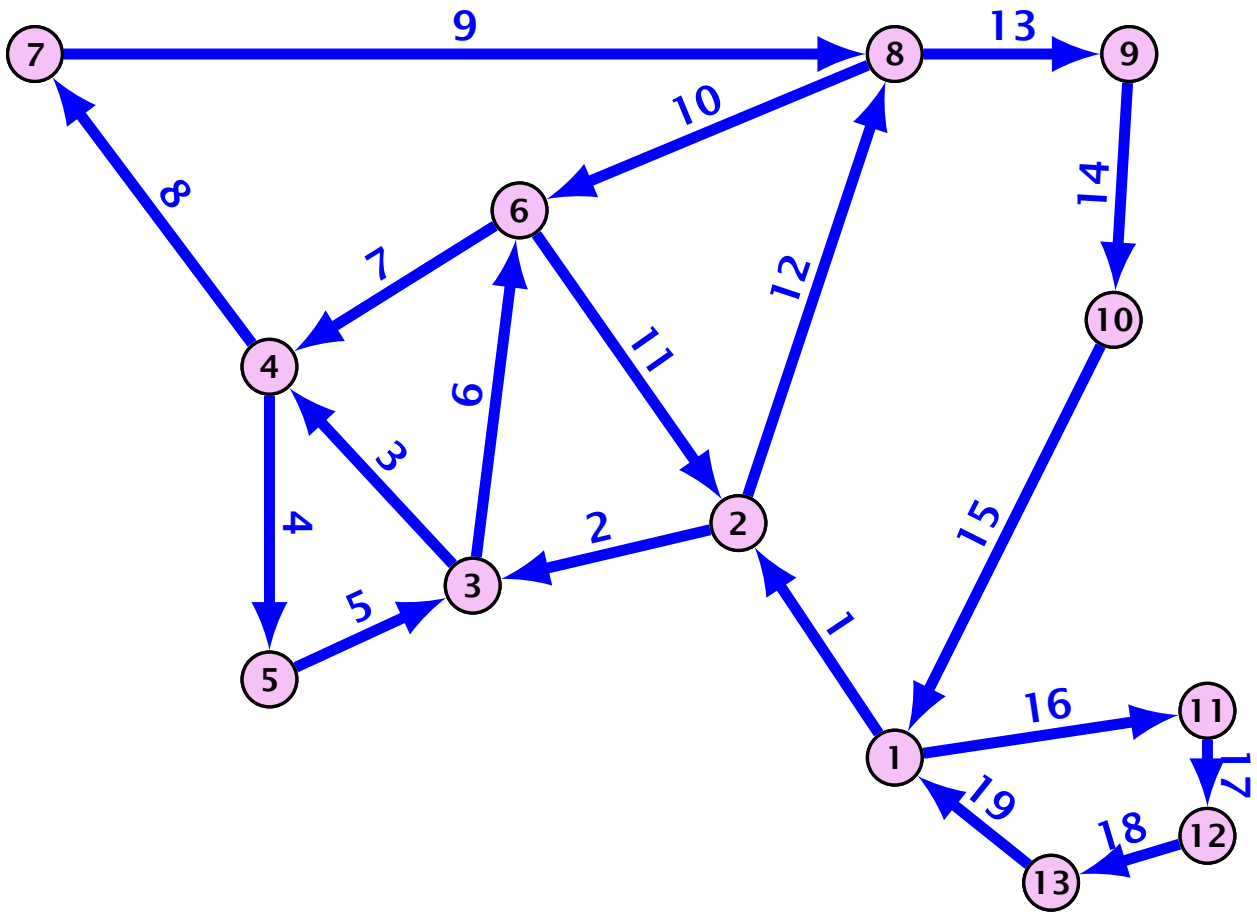

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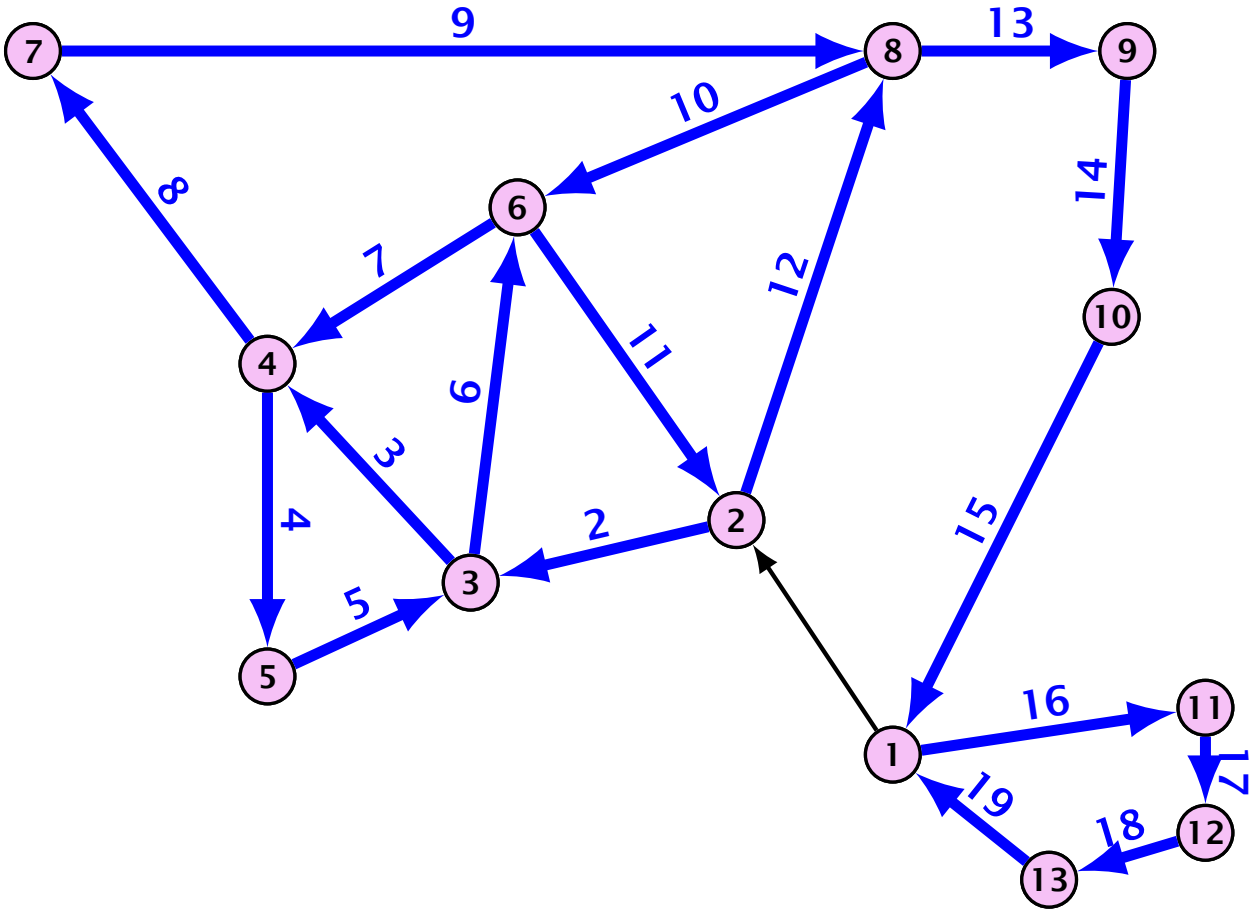

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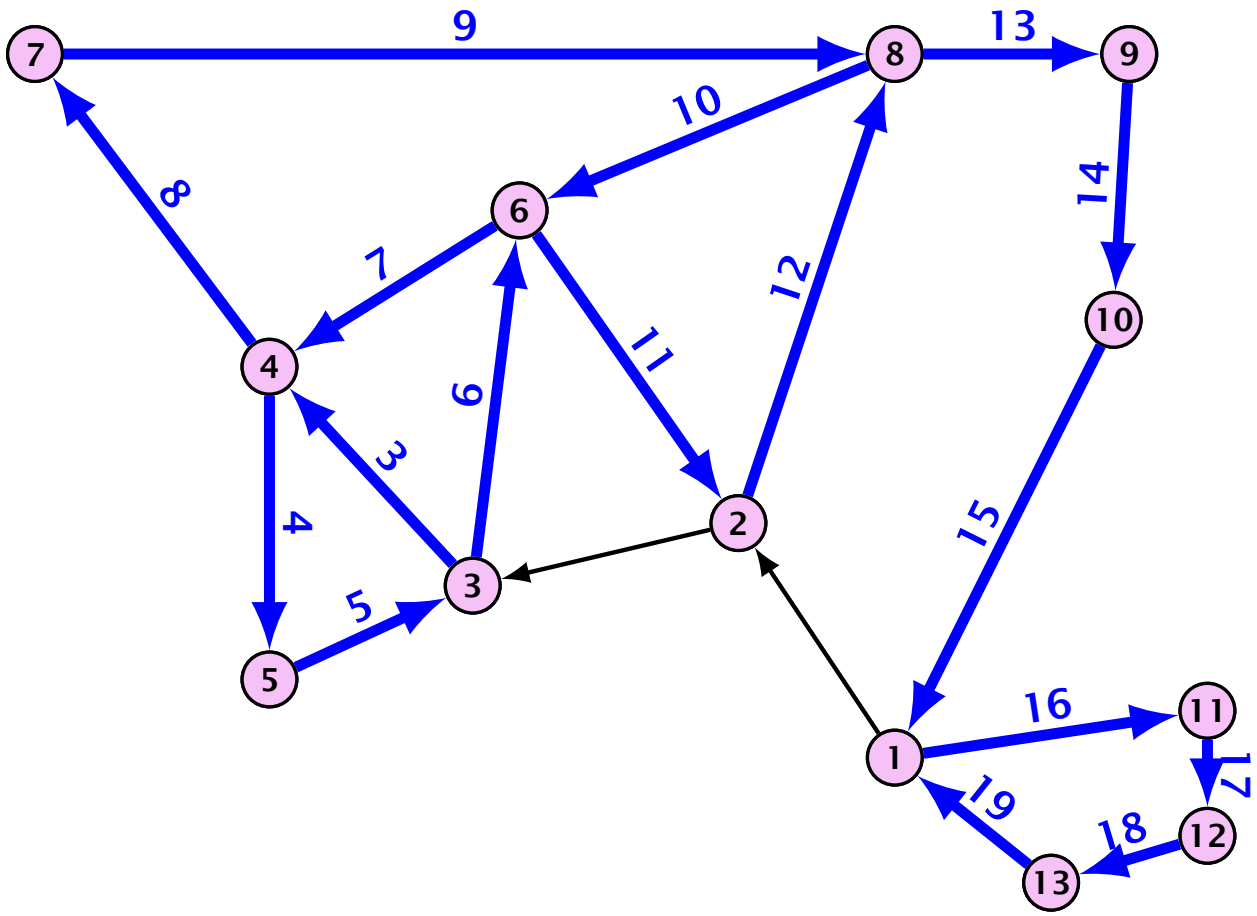

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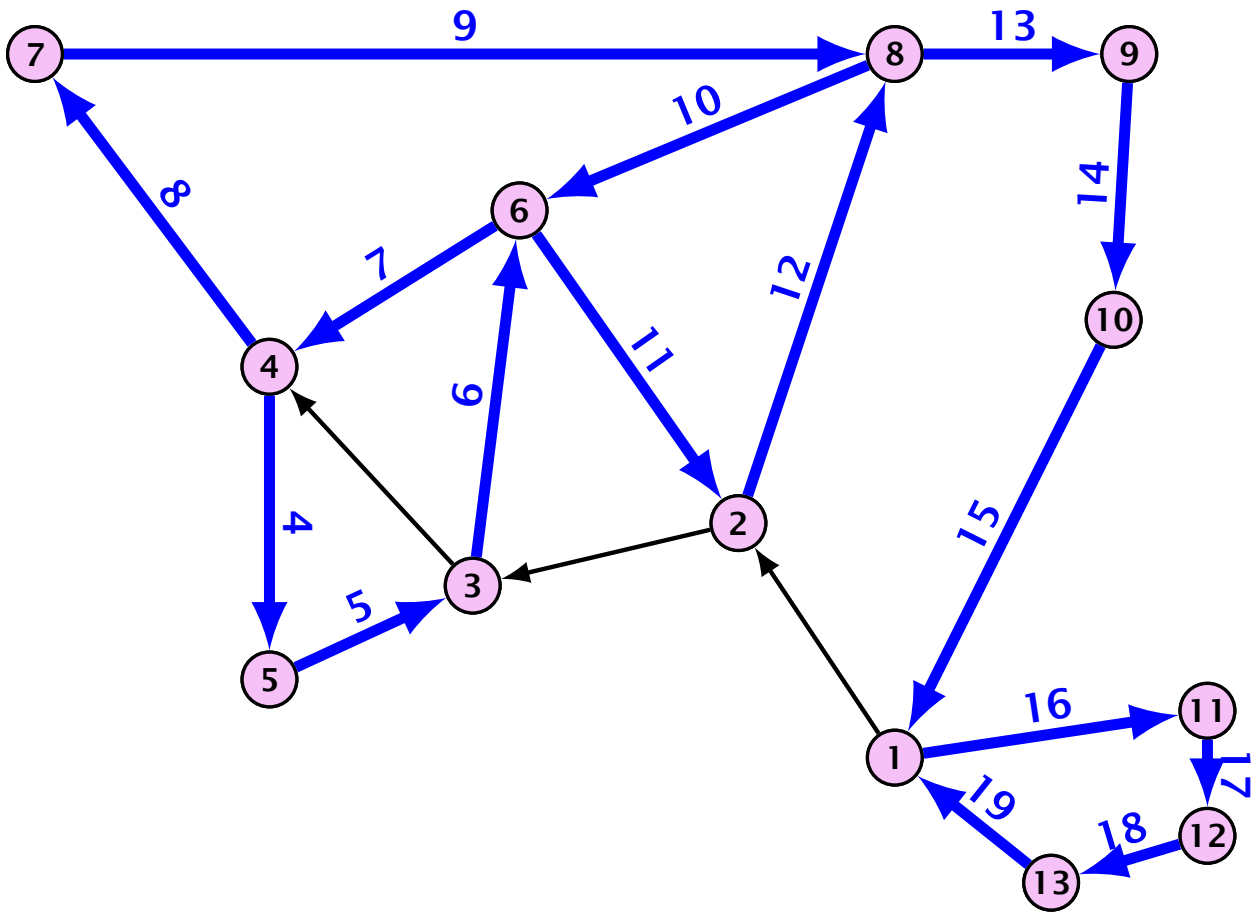

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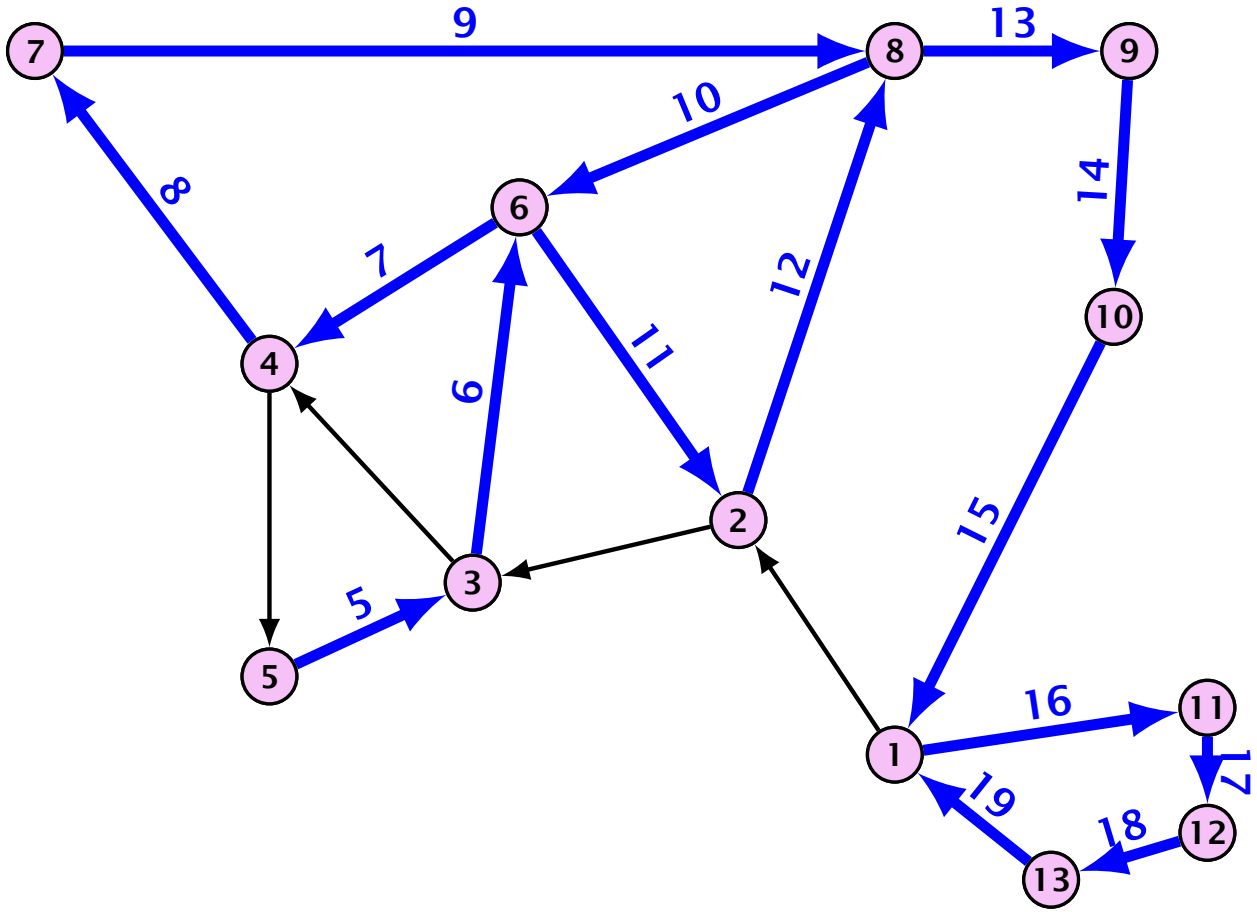

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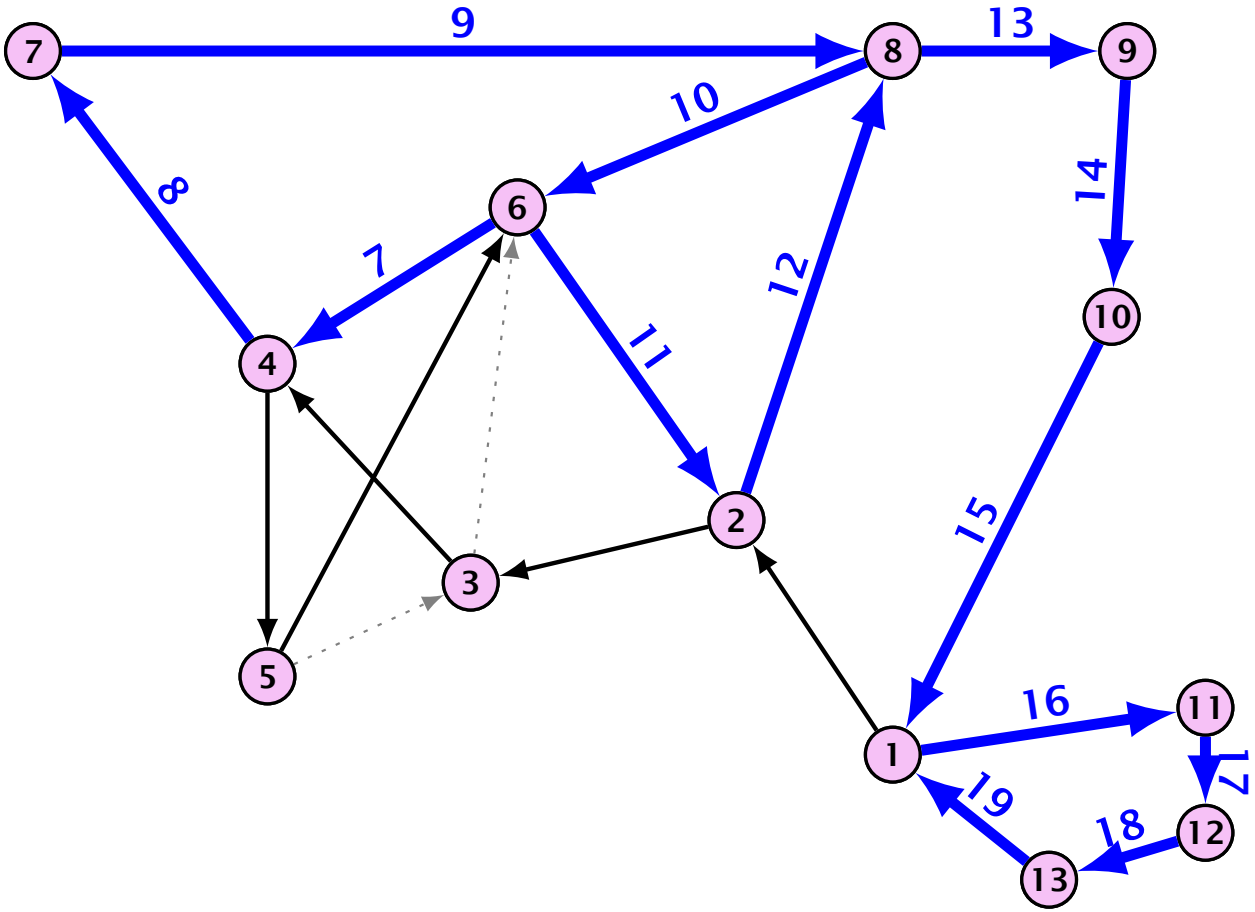

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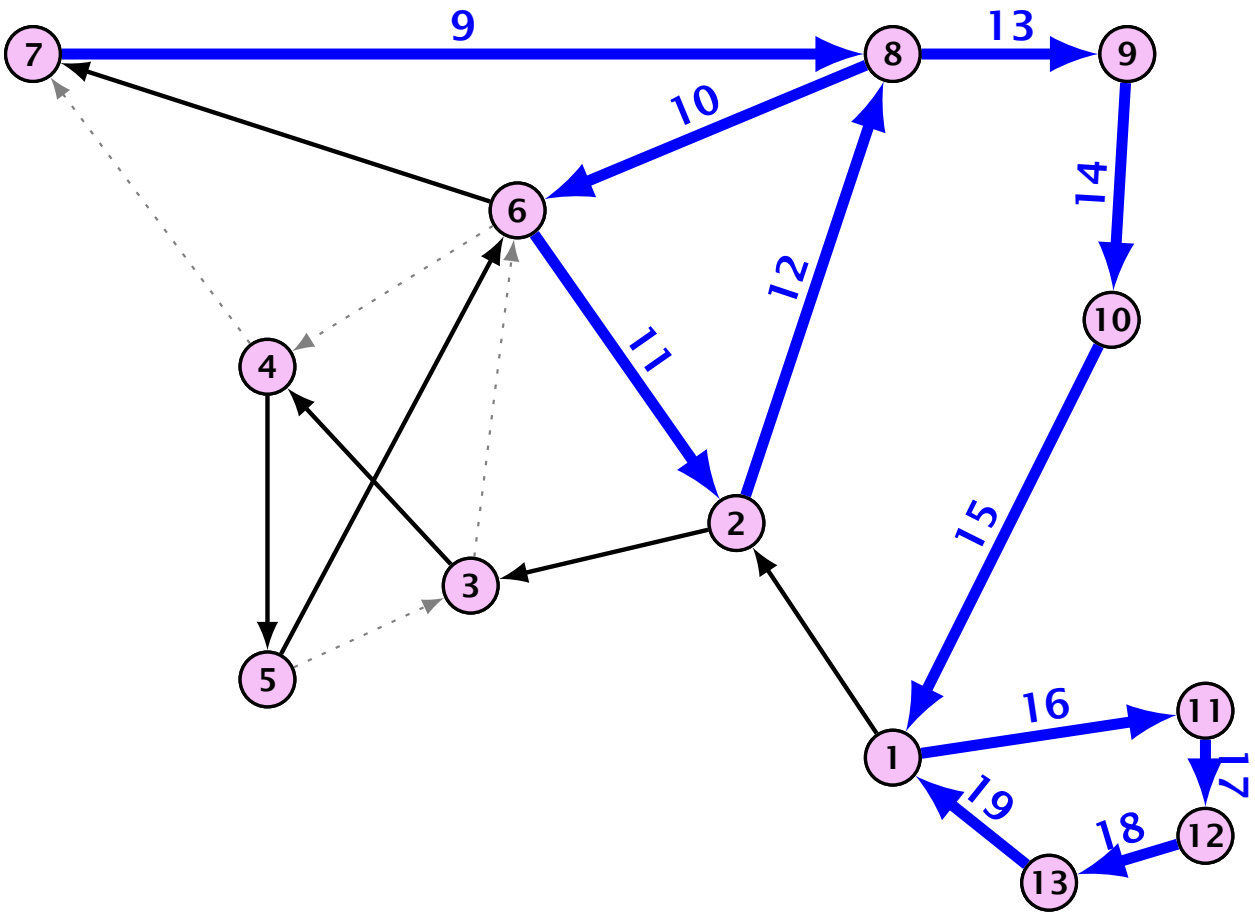

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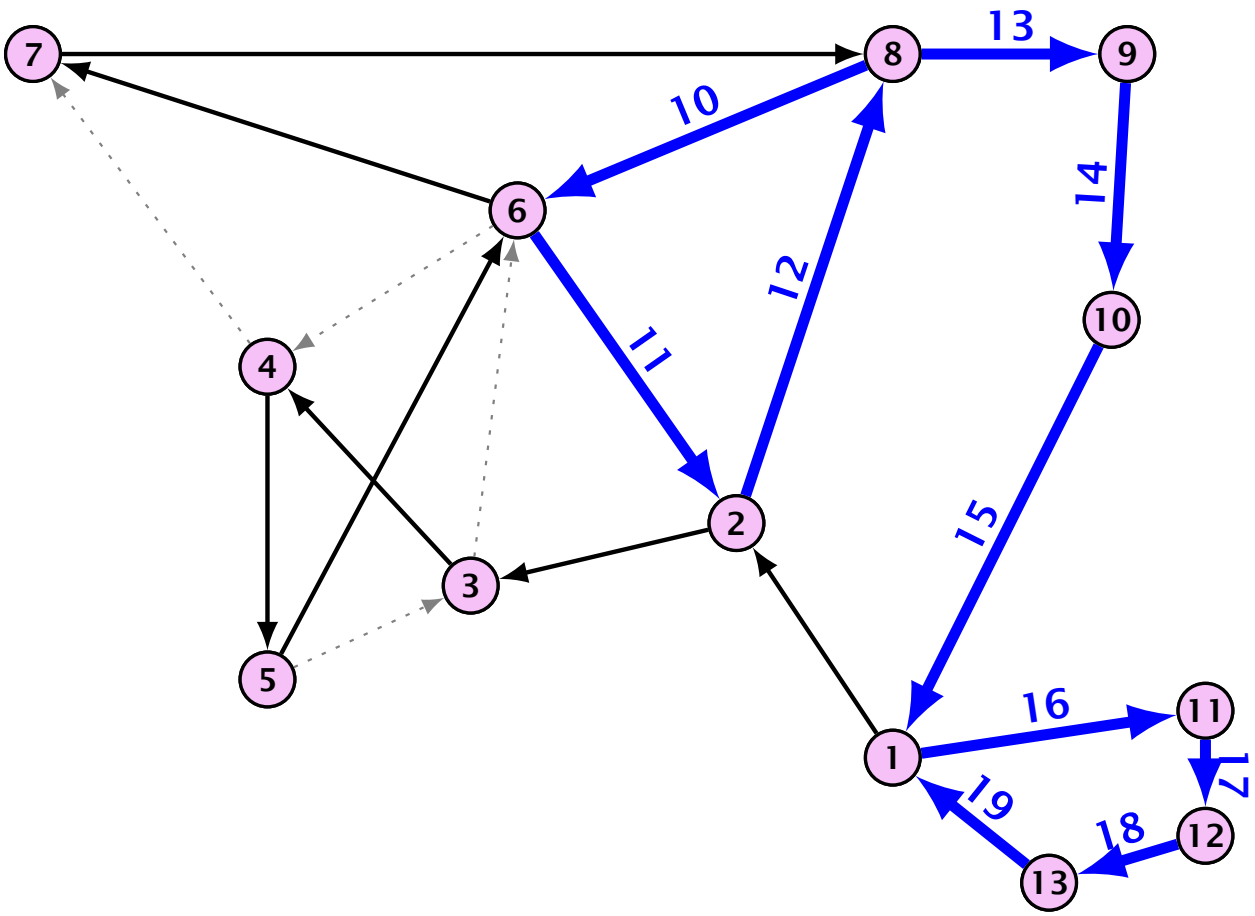

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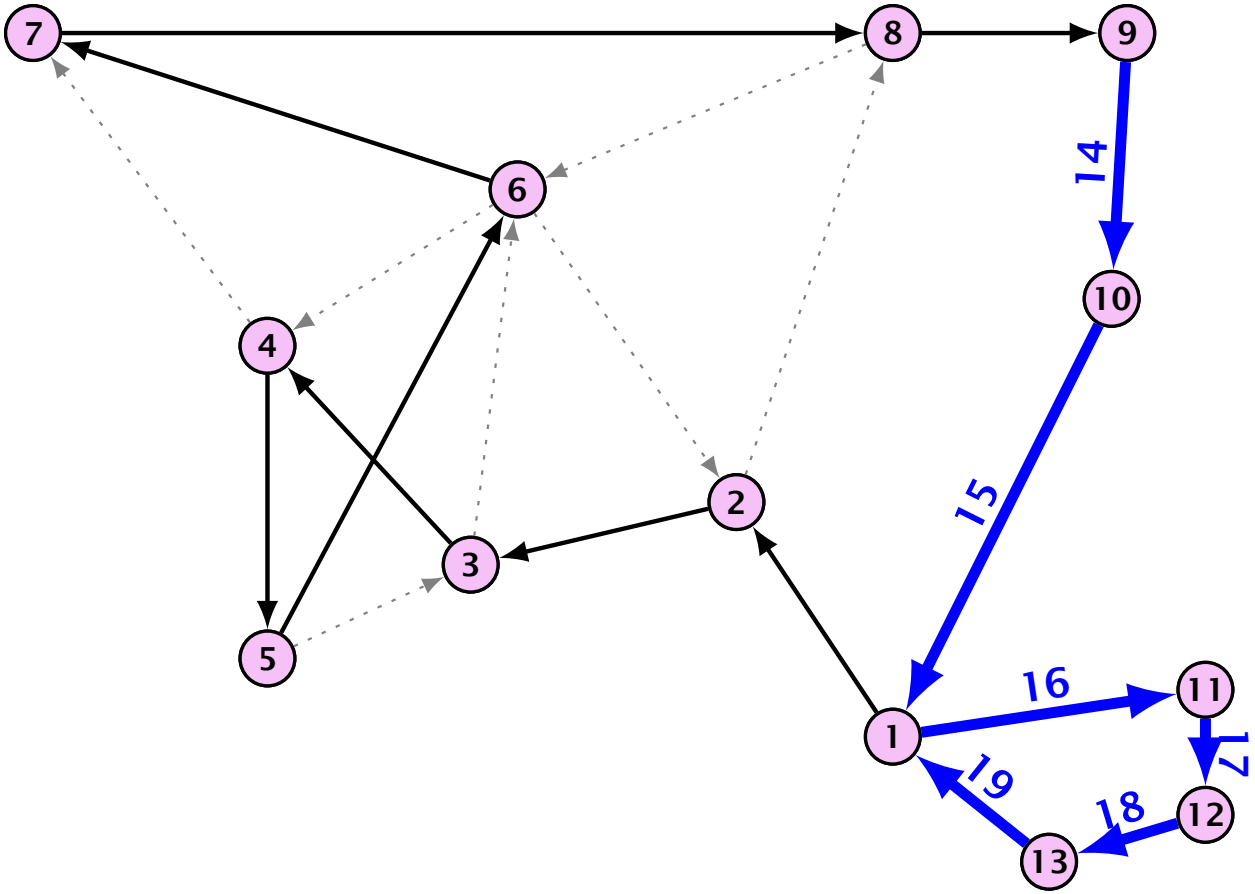

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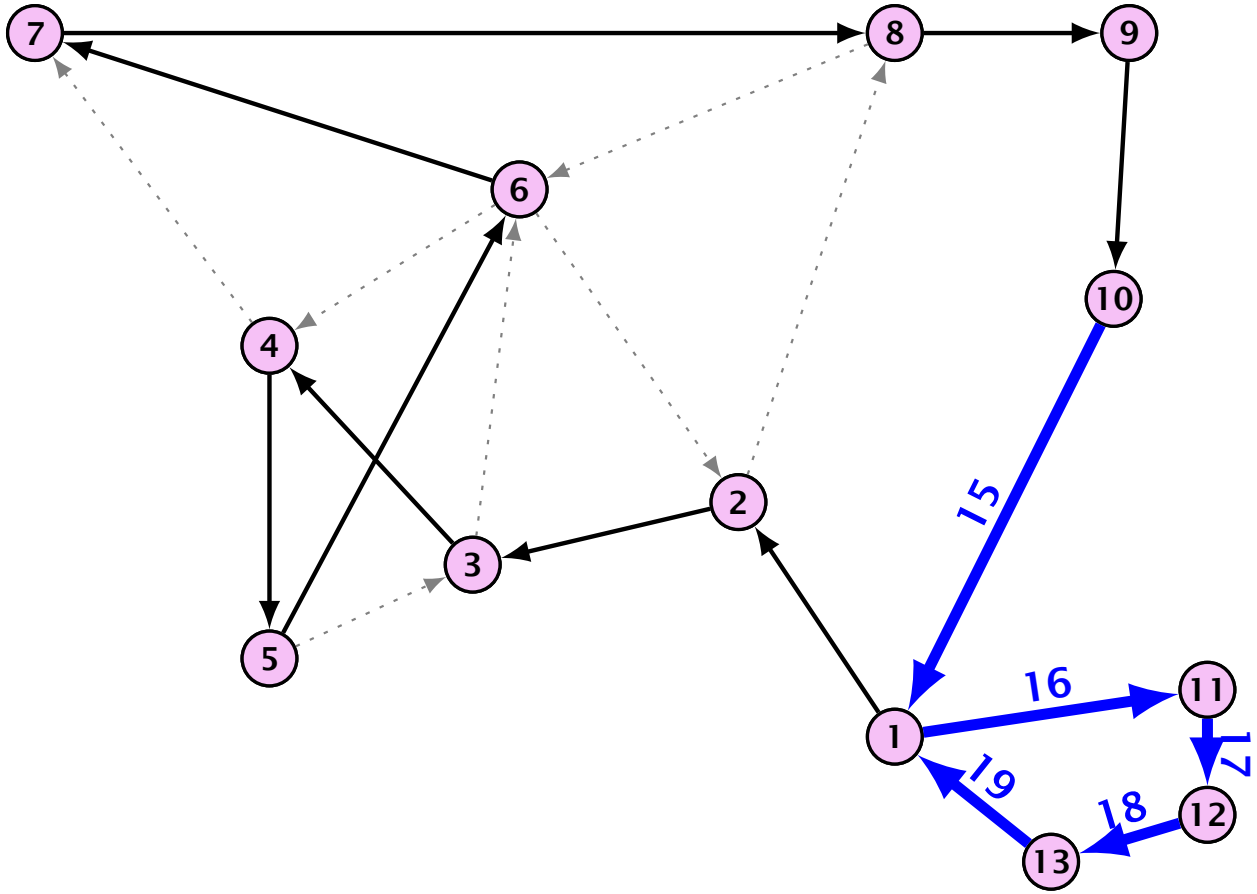

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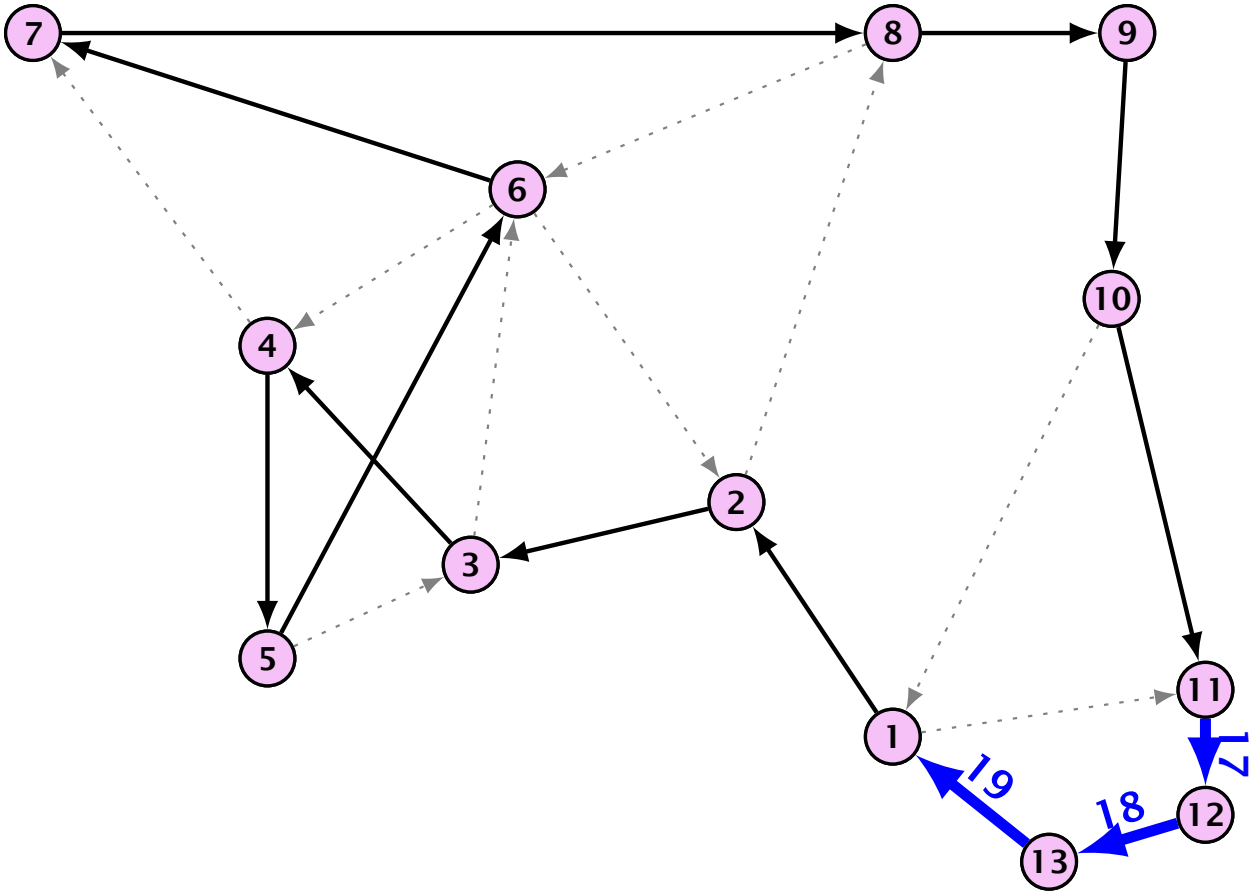

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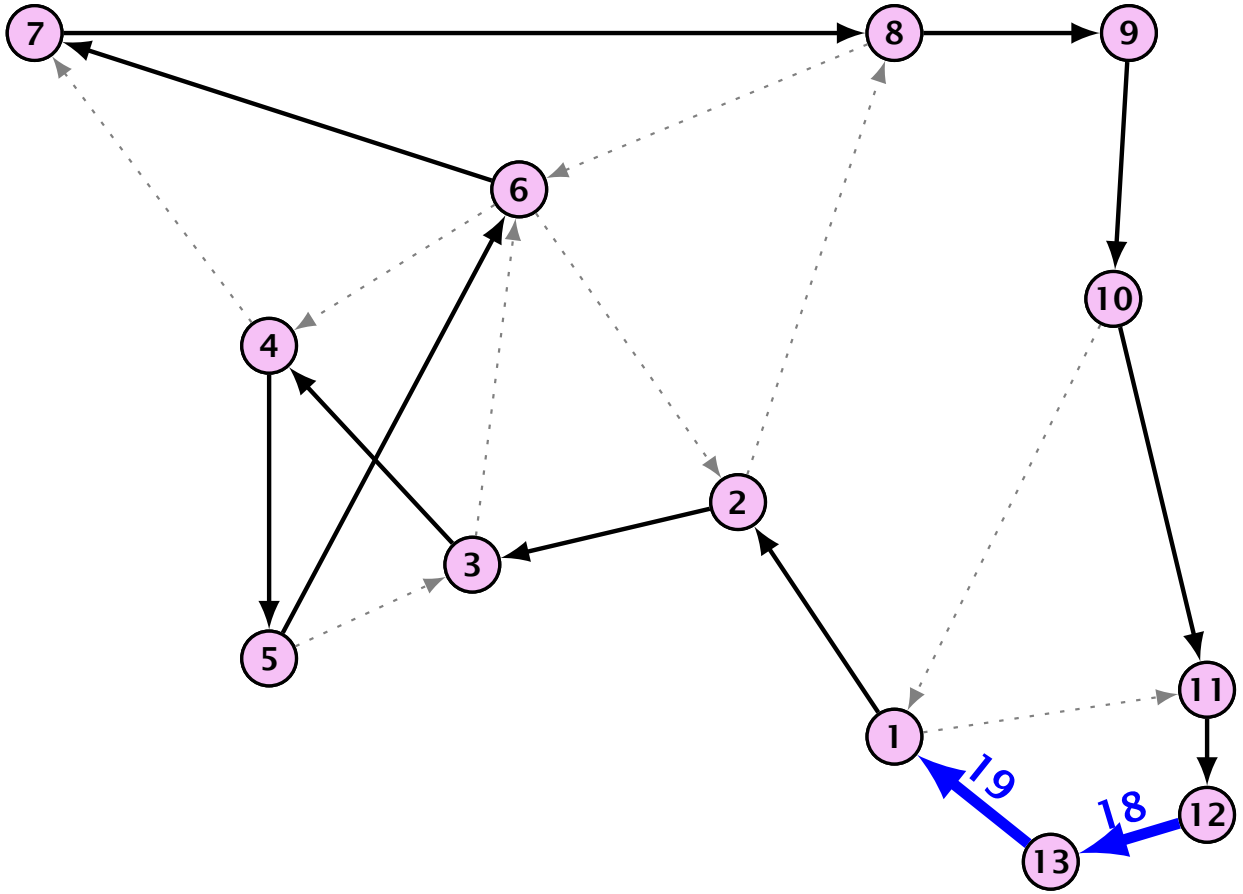

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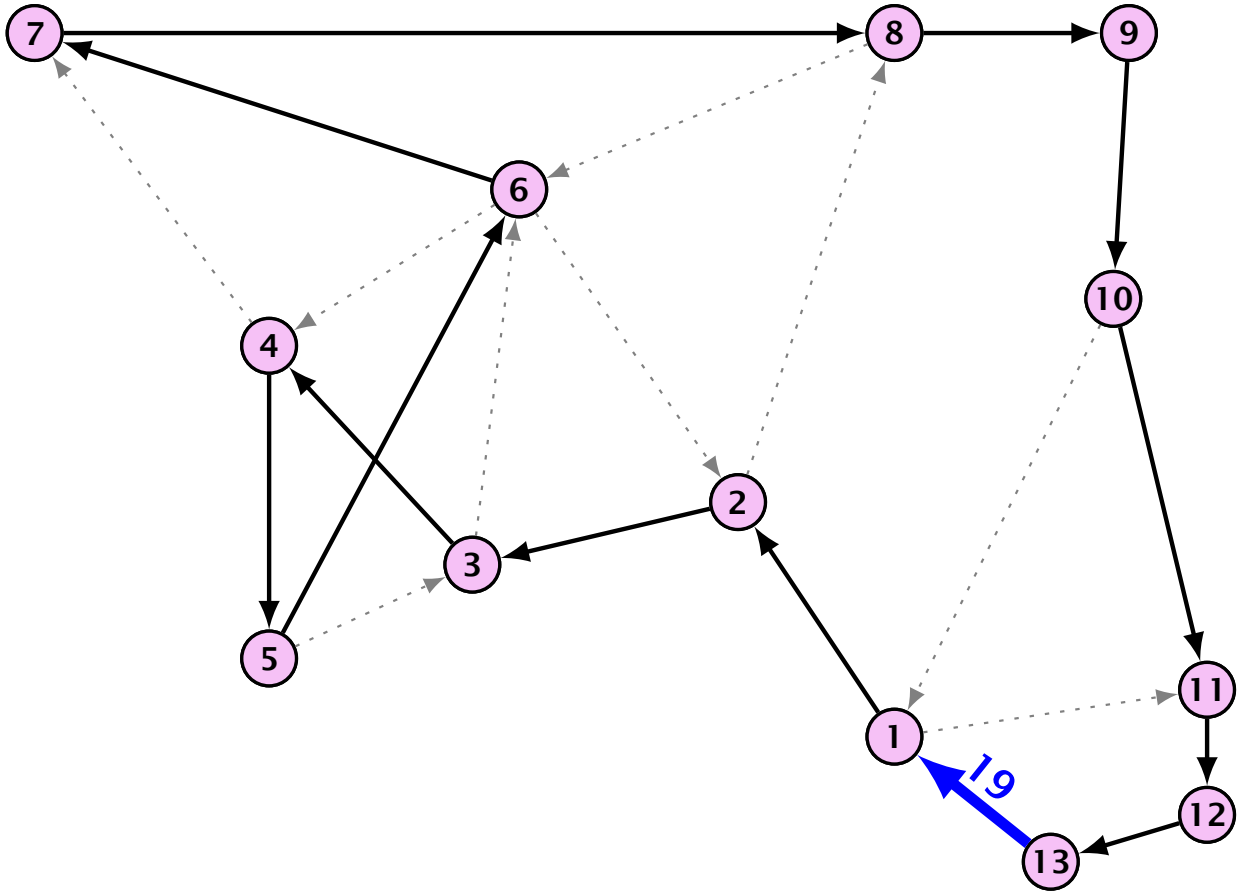

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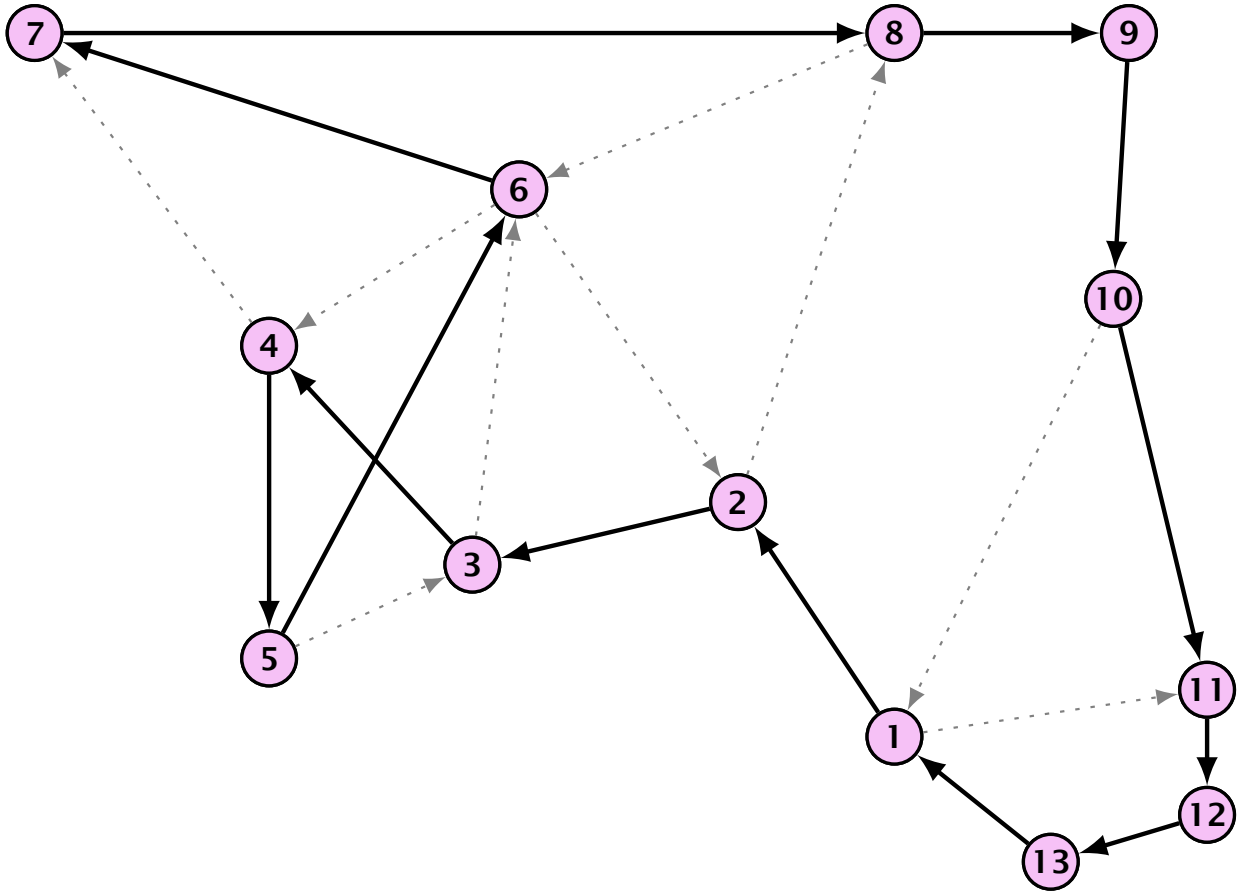

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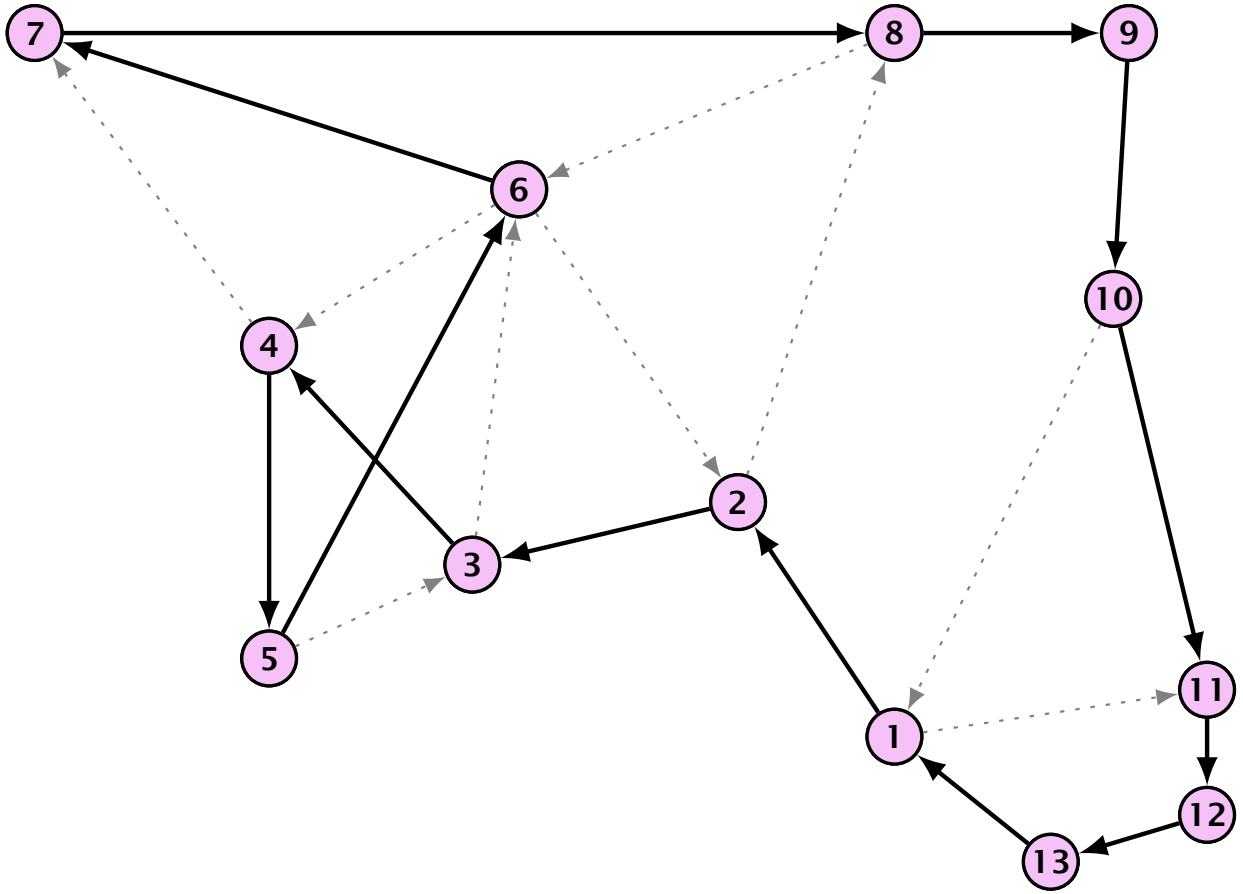

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Consider the following graph:

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- Duplicate all edges.

This graph is Eulerian, and the total cost of all edges is at most $2 \cdot \mathrm{OPT}_{\mathrm{MST}}(G)$.

Hence, short-cutting gives a tour of cost no more than $2 \cdot \mathrm{OPT}_{\mathrm{MST}}(G)$ which means we have a 2-approximation.

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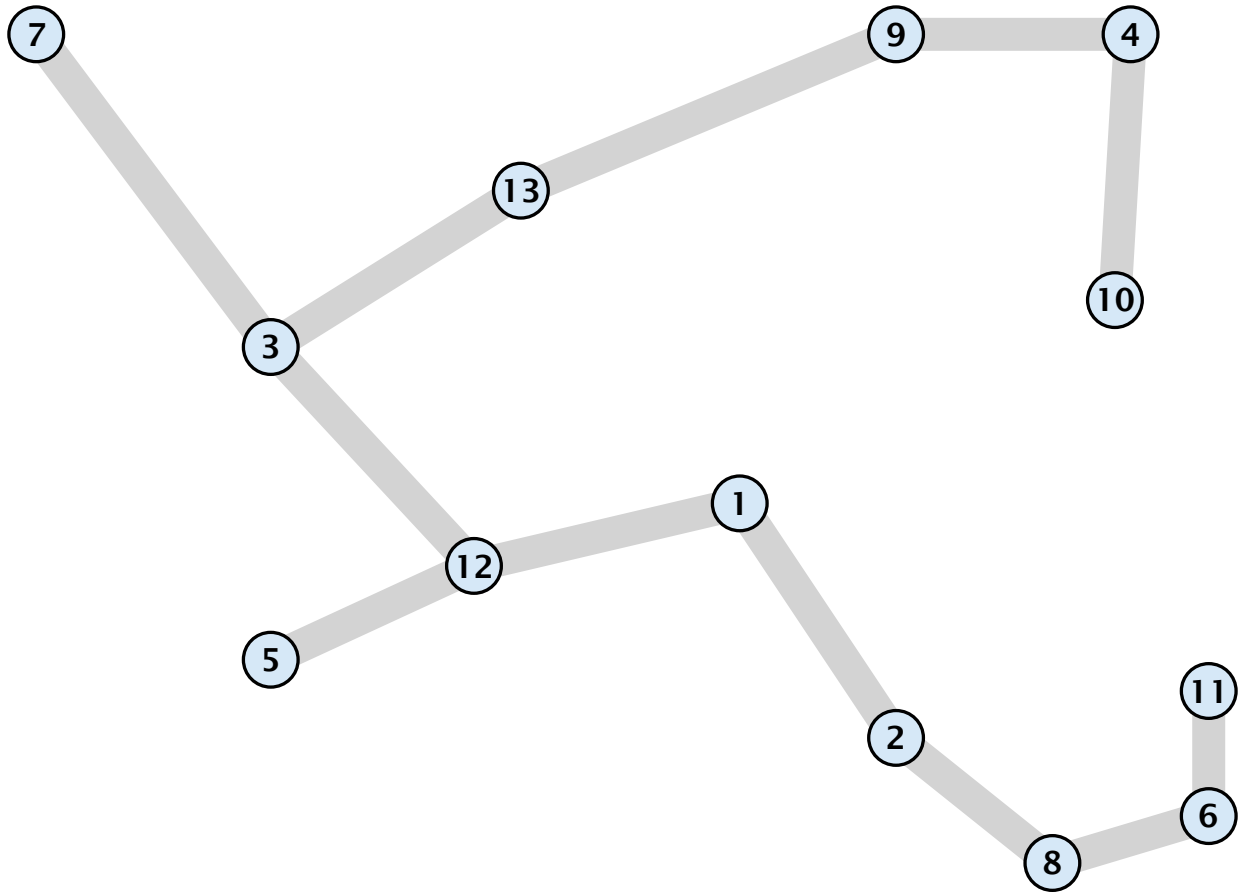

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Duplicating all edges in the MST seems to be rather wasteful.

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An optimal tour on the odd-degree vertices has cost at most $\mathrm{OPT}_{\mathrm{TSP}}(G)$.

However, the edges of this tour give rise to two disjoint matchings. One of these matchings must have weight less than $\mathrm{OPT}_{\mathrm{TSP}}(G)/2$.

Adding this matching to the MST gives an Eulerian graph with edge weight at most

$$\mathrm{OPT}_{\mathrm{MST}}(G) + \mathrm{OPT}_{\mathrm{TSP}}(G)/2 \leq \frac{3}{2}\mathrm{OPT}_{\mathrm{TSP}}(G) \ ,$$

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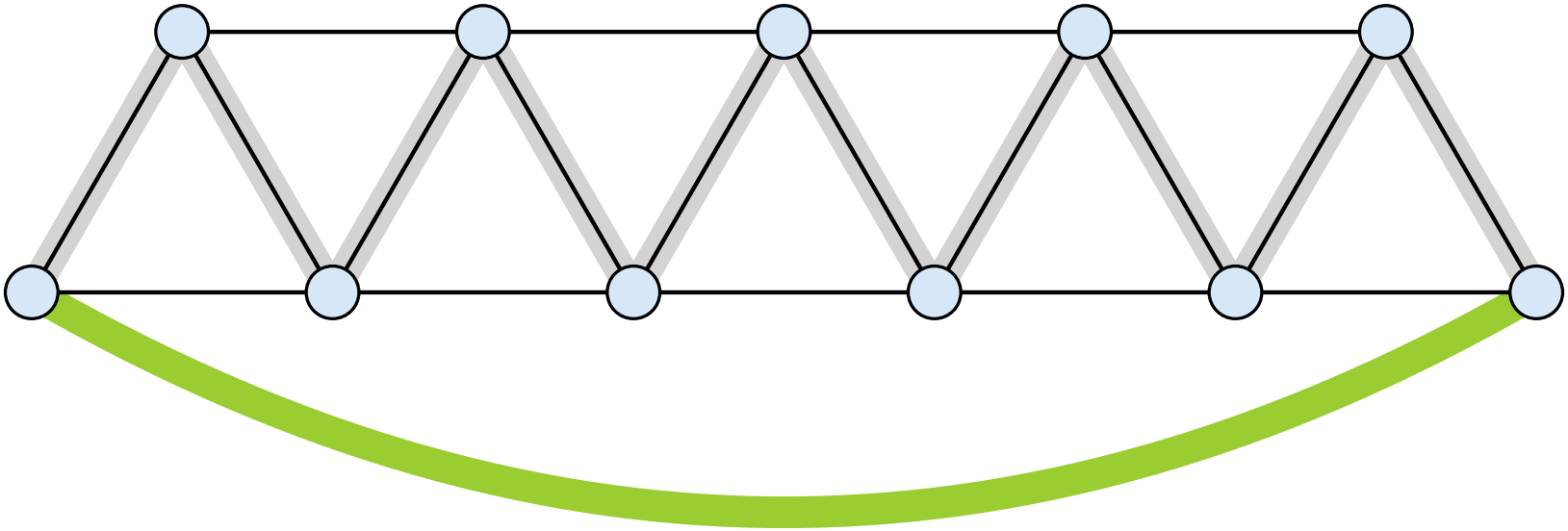
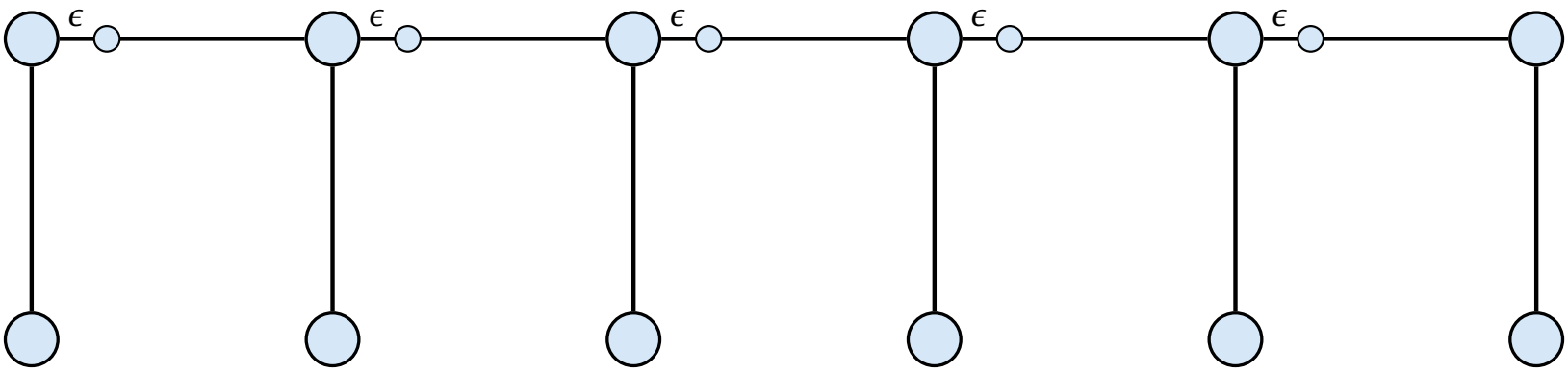
Christofides. Tight Example



- optimal tour: n edges.
- MST: $n-1$ edges.
- weight of matching $(n+1)/2 - 1$
- MST+matching $\approx 3/2 \cdot n$

Tree shortcutting. Tight Example



- edges have Euclidean distance.