7.4 Augmenting Data Structures

Suppose you want to develop a data structure with:

▶ **Insert(x):** insert element x.

▶ **Search(k):** search for element with key k.

▶ **Delete(x):** delete element referenced by pointer x.

▶ **find-by-rank(ℓ):** return the ℓ-th element; return "error" if the data-structure contains less than ℓ elements.

Augment an existing data-structure instead of developing a new one.

7.4 Augmenting Data Structures

Suppose you want to develop a data structure with:

- ▶ **Insert(x):** insert element x.
- ▶ **Search(k):** search for element with key k.
- ▶ **Delete(x):** delete element referenced by pointer x.
- ▶ **find-by-rank(ℓ):** return the ℓ-th element; return "error" if the data-structure contains less than ℓ elements.

Augment an existing data-structure instead of developing a new one.

7.4 Augmenting Data Structures

How to augment a data-structure

1. choose an underlying data-structure
2. determine additional information to be stored in the underlying structure
3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
4. develop the new operations

7.4 Augmenting Data Structures

How to augment a data-structure

1. choose an underlying data-structure
2. determine additional information to be stored in the underlying structure
3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.
4. develop the new operations

7.4 Augmenting Data Structures

How to augment a data-structure

1. choose an underlying data-structure

2. determine additional information to be stored in the underlying structure

3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.

4. develop the new operations

7.4 Augmenting Data Structures

How to augment a data-structure

1. choose an underlying data-structure

2. determine additional information to be stored in the underlying structure

3. verify/show how the additional information can be maintained for the basic modifying operations on the underlying structure.

4. develop the new operations

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

1. We choose a red-black tree as the underlying data-structure.

2. We store in each node v the size of the sub-tree rooted at v.

3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

1. We choose a red-black tree as the underlying data-structure.

2. We store in each node v the size of the sub-tree rooted at v.

3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.
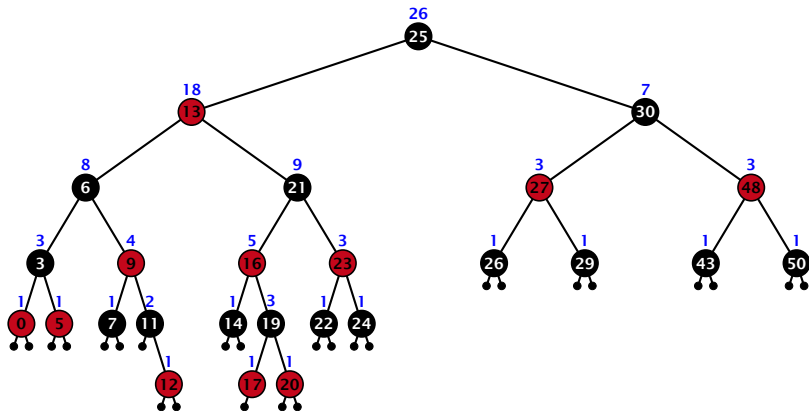
1. We choose a red-black tree as the underlying data-structure.

2. We store in each node v the size of the sub-tree rooted at v.

3. We need to be able to update the size-field in each node without asymptotically affecting the running time of insert, delete, and search. We come back to this step later...

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

4. How does find-by-rank work?
   Find-by-rank$(k) := $ Select$($root$, k)$ with

Algorithm 15 Select(x, i)

1: **if** $x = $ null **then return** error
2: **if** left$[x] \neq$ null **then** $r \leftarrow$ left$[x].$ size $+1$ **else** $r \leftarrow 1$
3: **if** $i = r$ **then return** x
4: **if** $i < r$ **then**
5:     **return** Select$($left$[x], i)$
6: **else**
7:     **return** Select$($right$[x], i - r)$

Select(x, i)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
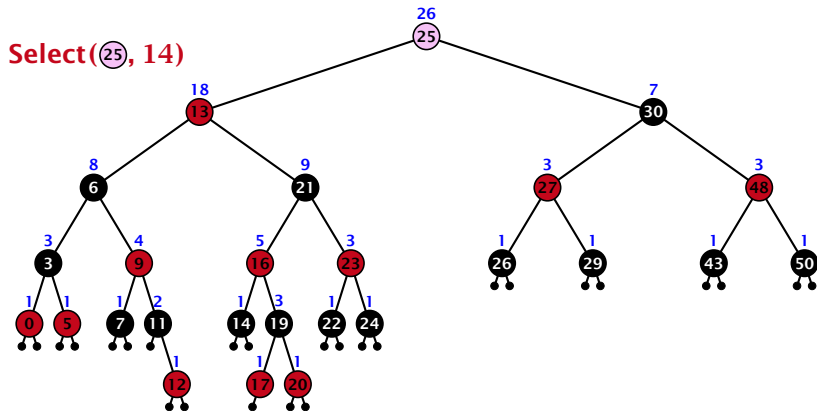- adjust the rank that you are searching for if you go right

Select(x, i)

Find-by-rank:

- ▶ decide whether you have to proceed into the left or right sub-tree
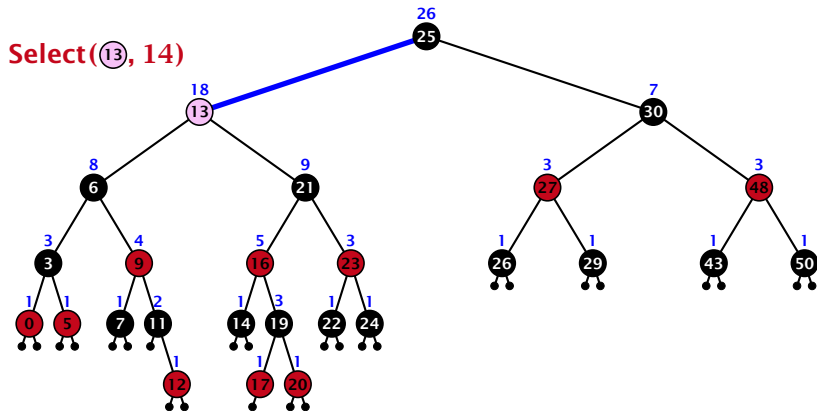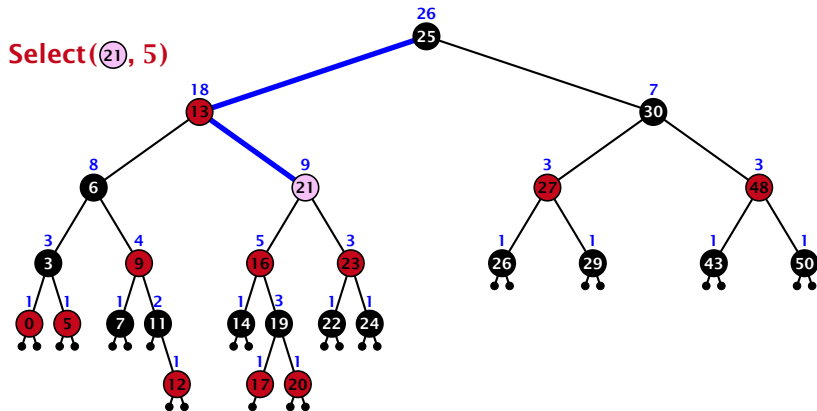- ▶ adjust the rank that you are searching for if you go right

Select(x, i)

Select($\textcircled{13}$, 14)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right

Select(x, i)

Find-by-rank:

- ▶ decide whether you have to proceed into the left or right sub-tree
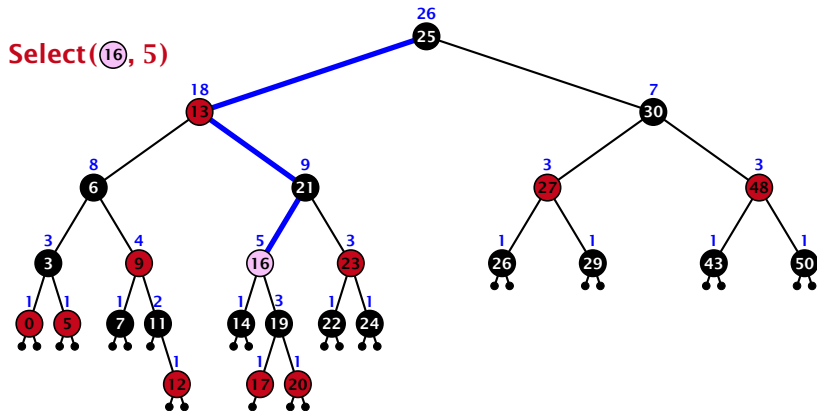- ▶ adjust the rank that you are searching for if you go right

Select(x, i)

Select(16, 5)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
- adjust the rank that you are searching for if you go right

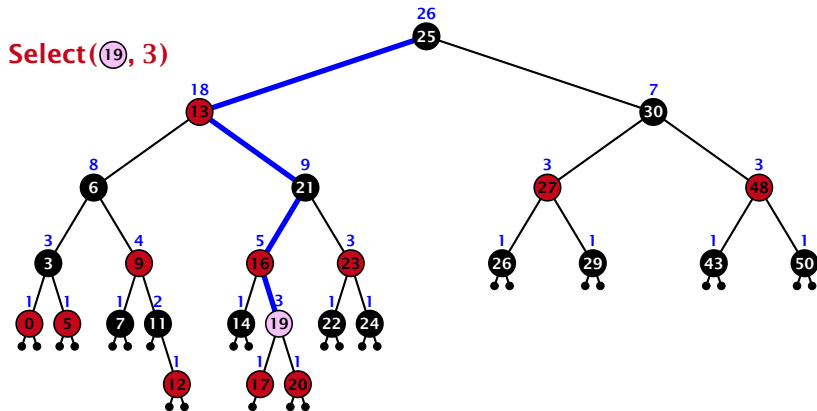# Select(*x*, *i*)

Select(⑲, 3)

Find-by-rank:

- decide whether you have to proceed into the left or right sub-tree
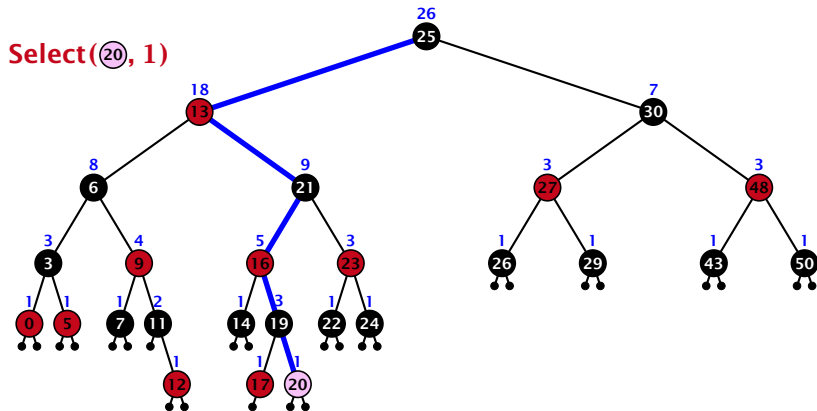- adjust the rank that you are searching for if you go right

Select(x, i)

Find-by-rank:

- ▶ decide whether you have to proceed into the left or right sub-tree
- ▶ adjust the rank that you are searching for if you go right

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

3. How do we maintain information?

Search(k): Nothing to do.

Insert(x): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete(x): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

   3. How do we maintain information?

Search(k): Nothing to do.

Insert(x): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete(x): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

3. How do we maintain information?

Search(k): Nothing to do.

Insert(x): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete(x): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

7.4 Augmenting Data Structures

Goal: Design a data-structure that supports insert, delete, search, and find-by-rank in time $\mathcal{O}(\log n)$.

   3. How do we maintain information?

Search(k): Nothing to do.

Insert(x): When going down the search path increase the size field for each visited node. Maintain the size field during rotations.

Delete(x): Directly after splicing out a node traverse the path from the spliced out node upwards, and decrease the size counter on every node on this path. Maintain the size field during rotations.

Rotations

The only operation during the fix-up procedure that alters the tree and requires an update of the size-field:
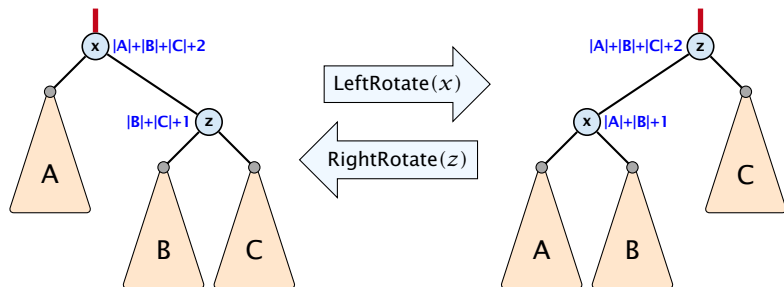
The nodes x and z are the only nodes changing their size-fields.

The new size-fields can be computed locally from the size-fields of the children.