Technische Universität München
Fakultät für Informatik
Lehrstuhl für Effiziente Algorithmen (LEA)
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Problem set 10
June 18, 2014

Online and approximation algorithms

Due June 25, 2014 before class!

Exercise 1 (Ice Cream Problem - 10 points)

Suppose you have a machine that is able to produce two flavors of ice cream, vanilla and chocolate. In state V the machine can make vanilla ice cream at the cost of \$1 per scoop, in state C the machine can make chocolate ice cream at the cost of \$2 per scoop. Changing the state of the machine costs \$1. In addition it is possible to make ice cream without using the machine at a cost of \$2 per scoop of vanilla and \$4 per scoop of chocolate.
When a customer arrives and requests a scoop of ice cream of a given flavor you have to decide whether to use the machine (which might force you into changing its state first) or to make the ice cream by hand.
Develop a $\frac{7}{6}$-competitive deterministic algorithm for this problem.

Exercise 2 (Chinese Postman Problem - 10 points)

We consider the Chinese Postman Problem (CPP). In this problem a postman wants to deliver his mail in the most efficient way. We model the territory he has to cover as a graph: Each intersection of two streets is a node and an edge is inserted between two intersections that are connected by a street. The weight of an edge is equal to the length of the street between the intersections.
The goal is to find the shortest tour that visits each edge of the resulting graph at least once.
Develop an algorithm that solves CPP.

Exercise 3 (k-server on the line - 10 points)

In the lecture we saw that the k-server problem on metric spaces that are trees is k-competitive. Show a similar result for metric spaces that are lines.

Exercise 4 (Traveling Salesman - 10 points)

Consider the following Binary Integer Program (BIP) where $G = (V, E)$ is the given graph and $c_{i,j}$ is the weight of the edge between vertex i and vertex j:

$$
\begin{array}{llll}
\min & \sum_{j=1}^{n} \sum_{i=1}^{n} c_{i,j} x_{i,j} & & \\
s.t. & \sum_{i:i \neq j} x_{i,j} & = 1 & \text{for } j = 1, ..., n \\
 & \sum_{j:i \neq j} x_{i,j} & = 1 & \text{for } i = 1, ..., n \\
(*) & \sum_{i \in S} \sum_{j \in S} x_{i,j} & \leq |S| - 1 & \text{for } \emptyset \subset S \subset V \\
 & x \in \mathbb{B}^{n \times n} & &
\end{array}
$$

(a) Argue that the above BIP describes an optimal solution to the Traveling Salesman Problem.

(b) What happens when $(*)$ is removed from the BIP?

(c) Remove $(*)$ from the BIP and replace it with an alternative inequality s.t. the BIP still describes an optimal solution to the Traveling Salesman Problem.