
Automata and Formal Languages

Due November 25, 2014 before class!

Exercise 1 (Pattern Matching II - 10 points)

Construct an NFA, a DFA and a lazy DFA for the pattern *mammal*.

Exercise 2 (Pattern Matching III - 10 points)

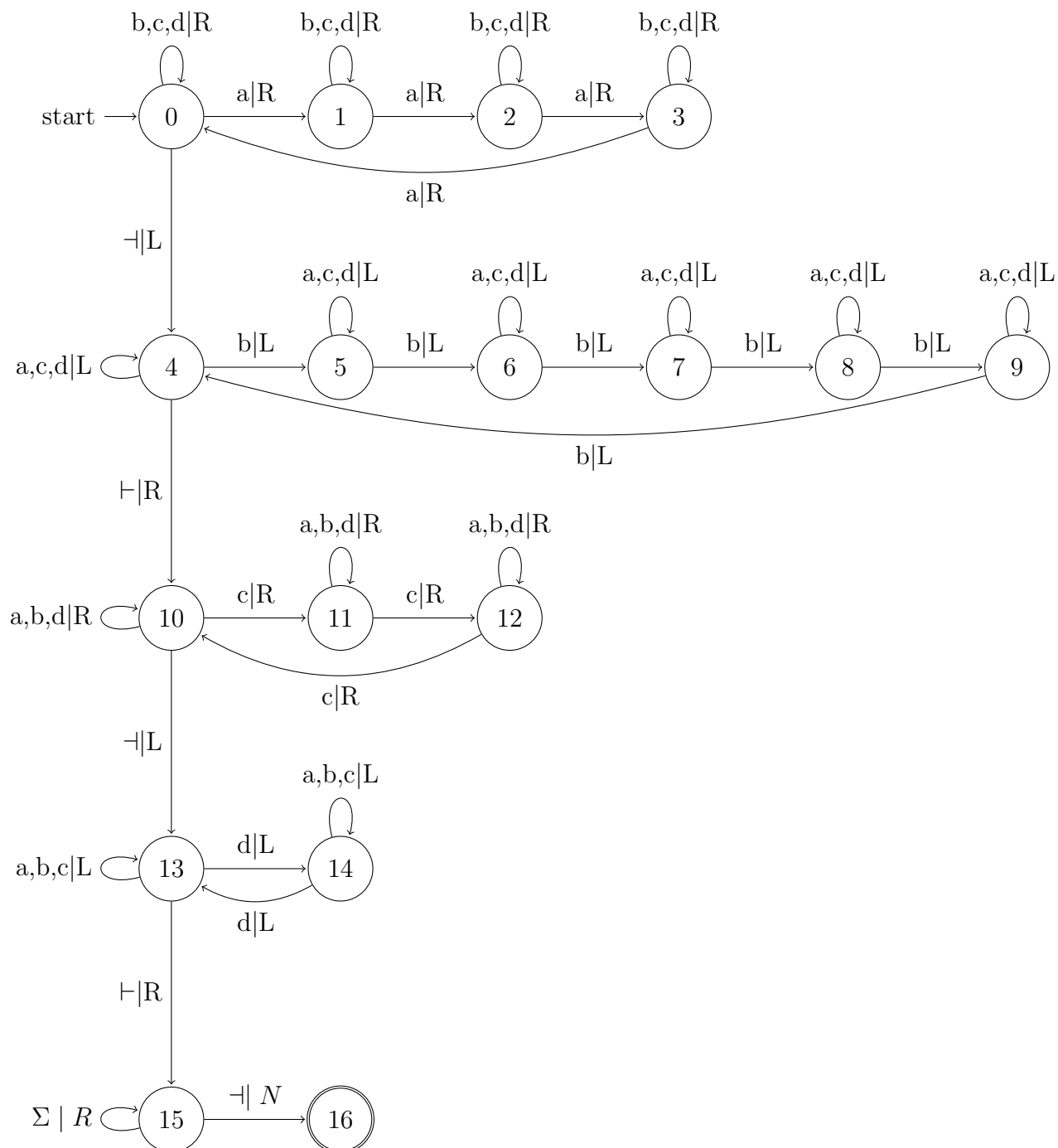
We have shown that lazy DFAs for a word pattern may need more than n steps to read a text of length n , but not more than $2n + m$, where m is the length of the pattern.

Let B_p be a NFA for the pattern p and C_p be a lazy NFA for the pattern p . Find a text t and a word pattern p such that the run of B_p on t takes at most n steps and the run of C_p on t takes at least $2n - 1$ steps.

Exercise 3 (2-DFA I - 10 points)

2-DFAs (and similarly 2-NFAs) are an extension of lazy DFAs, where in addition to staying put and moving right, the head is also allowed to move left. The transition function is then of the type $Q \times \Sigma \rightarrow P(S) \times \{-1, 0, 1\}$. For better readability we may refer to $-1, 0$ and 1 as L, N and R. We say that a 2-DFA accepts a word if it stops on the $n + 1$ -st position of the input string while being in a final state.

- (a) We assume that on the left end of the input string we have the symbol \vdash , on the right end the symbol \dashv . Describe the language the following 2-DFA recognizes.



(b) Give a 2-DFA for the language $r = (0 + 1)^*1(0 + 1)^k$ which has $\mathcal{O}(k)$ states.

Exercise 4 (2-DFA II - 10 points)

(a) Prove the following Lemma:

Let $A = (\Sigma, S, S_0, \rho, F)$ be a 2-DFA and $w = a_0 \dots a_n \in \Sigma^*$. A does not accept if and only if there is a sequence T_0, \dots, T_{n+1} of subsets of S such that

- $S_0 \subseteq T_0$
- $T_{n+1} \cap F = \emptyset$
- For $0 \leq i \leq n$, if $s \in T_i$, $(s', k) \in \rho(s, a)$ and $i + k > 0$ then $s' \in T_{i+k}$.

(b) Use the lemma from the previous exercise to prove that 2-DFA are equally powerful as usual DFAs.

Hint: Construct an automaton that recognizes $\overline{\mathcal{L}(A)}$