Technische Universität München
Fakultät für Informatik
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Winter Semester 2014/15
Problem Set 1
October 13th, 2014

Parallel Algorithms

Due date: October 20th, 2014 **before** *class!*

Problem 1 (10 Points)

Consider an array with n distinct elements. The task is to search the array for a particular element x. We know that a sequential algorithm will always have a worst-case running time of n steps. Derive an efficient parallel algorithm for this task on an EREW (exclusive read / exclusive write) PRAM with $p \leq n$ processors.

Problem 2 (10 Points)

Prove Lemma 4 from the lecture: Given the DAG model, a schedule with length $\mathcal{O}\left(T_1(n)/p + T_\infty(n)\right)$ can be found easily.

Problem 3 (10 Points)

Given an integer sequence $(a_0, \ldots, a_n)$, the prefix sums are the elements of the integer sequence $(s_0, \ldots, s_n)$ with

$$\begin{aligned} s_0 &= a_0 \\ s_1 &= a_0 + a_1 \\ s_2 &= a_0 + a_1 + a_2 \\ &\ldots \end{aligned}$$

Assume a quadratic mesh with n processors, where you may assume that $\sqrt{n}$ is an integer. Show how to compute the prefix sums of the elements in the mesh processors in $\mathcal{O}\left(\sqrt{n}\right)$ time. What are the corresponding work requirement and speedup of your algorithm?

Problem 4 (10 Points)

Let A be a Boolean array of size n.

1. Develop an $\mathcal{O}\left(1\right)$ time CRCW PRAM algorithm to find the smallest index k such that $A(k) = 1$. The total number of operations must be $\mathcal{O}\left(n\right)$. *Hint*: Use a $\sqrt{n}$ divide-and-conquer strategy.

2. How fast can you solve this problem on the CREW PRAM? Your algorithm must use $\mathcal{O}\left(n\right)$ operations.