Technische Universität München
Fakultät für Informatik
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Winter Semester 2014/15
Problem Set 02
October 20th, 2014

Parallel Algorithms

Due date: October 27th, 2014 **before** *class!*

Problem 1 (20 Points)

Let A be an array of n integers in the range of $\{1, \ldots, \log n\}$.

1. Calculate the number of occurrences of an integer i in A for all $i \in \{1, \ldots, \log n\}$ in $\mathcal{O}(\log n)$ time using $\mathcal{O}(n)$ operations.
2. Calculate for every entry s with $A[s] = i$ the number of occurrences of integer i that come before s, i.e. with index lower than s, with a work requirement of $\mathcal{O}(n)$.
3. Sort the array A using $\mathcal{O}(\log n)$ time and a total of $\mathcal{O}(n)$ operations.

Problem 2 (10 Points)

Given a set $S = \{p_1, \ldots, p_n\}$ of n points in the plane, each represented by its (x, y) coordinates, the *planar convex hull* of S is the smallest convex polygon containing all the n points of S. A polygon Q is convex if for any two points p, q in Q, the line segment with endpoints p and q lies entirely in Q. The convex hull problem is to determine the ordered (say, clockwise) list $CH(S)$ of the points of S that define the boundary of the convex hull of S.

Give an algorithm for the convex hull problem that runs in $\mathcal{O}\left(\log^2 n\right)$ time and uses $\mathcal{O}(n \log n)$ operations. You may use the fact that sorting n numbers can be done in $\mathcal{O}(\log n)$ time on an EREW PRAM using $\mathcal{O}(n \log n)$ operations.

Hint: Divide the convex hull problem into the two subproblems for the *upper hull* and the *lower hull*, meaning the upper and lower part of the convex hull, respectively. Use divide-and-conquer (using a recurrence relation for the time and work requirements) to solve these subproblems.

Problem 3 (10 Points)

Show that the problem of finding the ordered list of vertices defining the convex hull of n points in the plane requires $\Omega(n \log n)$ operations.

Hint: Consider the set of points (x_i, x_i^2), where $1 \leq i \leq n$.