Prof. Dr. Susanne Albers
Dr. Dimitrios Letsios
Lehrstuhl Theoretische Informatik
Fakultät für Informatik
Technische Universität München

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Randomized Algorithms

Exercise Sheet 9

Due: December 22, 2014

Exercise 1: (10 points)

- Compute the moment generating function of a Binomial random variable with parameters (n, p). *Hint: Use the binomial identity* $(x + y)^n = \sum_{j=0}^{n} \binom{n}{j} x^j y^{n-j}$.
- Let X and Y be independent Binomial random variables with parameters (n, p) and (m, p), respectively. Compute the moment generating function of the random variable $Z = X + Y$. What can you say about Z?

Exercise 2: (10 points)
Consider a game in which we have $\frac{2}{3}$ probability of winning. We play 30 such games independently and we consider the probability of winning 5 or fewer games? Compute upper bounds (numerical values) on this probability by using Chebyshev's inequality and an appropriate Chernoff bound.

Exercise 3: (10 points)
Consider a **BPP** algorithm which has error probability $\frac{1}{2} - \frac{1}{p(n)}$, where $p(n)$ is a polynomial of the input size n. By using a Chernoff bound for the tail of the binomial distribution, show that a polynomial number of indepedent repetitions of the algorithm are sufficient to reduce the error probability to $\frac{1}{2^n}$.

Exercise 4: (10 points)
We throw n balls uniformly at random into n bins. By using a Chernoff bound, show that the probability that a bin contains more than $\frac{\ln n}{\ln \ln n}$ balls is at most $\frac{1}{n}$ for large n.