
Complexity Theory

Due date: June 22, 2015 before class!

Problem 1 (10 Points)

Show the details of the proof of Theorem 5.6. from the lecture:

For every $i \geq 1$, if $\Sigma_i^p = \Pi_i^p$, then $\mathbf{PH} = \Sigma_i^p$.

Problem 2 (10 Points)

Recall the definition of alternating Turing machines (ATM) with control states partitioned into sets Q_\forall and Q_\exists , and the corresponding class \mathbf{AP} .

1. Show that a language $L \in \mathbf{AP}$ decided by an *existential* ATM (i.e. $Q_\forall = \emptyset$) is in \mathcal{NP} .
2. Show that a language $L \in \mathbf{AP}$ decided by an *universal* ATM (i.e. $Q_\exists = \emptyset$) is in $\text{co}\mathcal{NP}$.
3. Show that $\mathbf{AP} = \text{co}\mathbf{AP}$.
4. Show that \mathbf{PSPACE} is contained in \mathbf{AP} by showing that $\text{TQBF} \in \mathbf{AP}$.

Problem 3 (10 Points)

Prove $\mathbf{AL} = \mathcal{P}$.