Complexity Theory

Due date: June 29, 2015 **before** class!

Problem 1 (10 Points)

Find oracles $A, \ldots, E$ such that the following hold:

1. $\mathcal{P}^A = \textbf{PSPACE}^A$,

2. $\textbf{PH}^B = \textbf{PSPACE}^B$,

3. $\textbf{PH}^C = \mathcal{P}^C$,

4. $\textbf{L}^D = \textbf{PSPACE}^D$,

5. $\textbf{L}^E = \mathcal{P}^E$.

Problem 2 (10 Points)

Consider the language $A = \{a^n b^n : n \geq 0\}$, which is in $\mathcal{P}$. Furthermore, consider a reduction VERTEX COVER $\preceq_m^p A$

$$f(G, k) = \begin{cases} aabb & \text{if } G \text{ has a vertex cover of size } k \\ aab & \text{otherwise} \end{cases}.$$

Since VERTEX COVER is $\mathcal{NP}$-complete, we get that $\mathcal{P} = \mathcal{NP}$. Where is the flaw in this "proof"?

Problem 3 (10 Points)

Describe a decidable language in $\mathcal{P}_{/poly}$ that is not in $\mathcal{P}$.

Problem 4 (10 Points)

Consider the problem of checking a boolean formula's syntactical correctness. Show that this problem can be decided in log-space, even if we have no precedence relation between the boolean operators and force precedence behavior with parentheses, e.g. $(x \wedge y) \vee (\overline{z} \wedge x) \vee \overline{y} \vee z$ is a valid formula, as is $(x \wedge (y \vee \overline{z}) \wedge x) \vee \overline{y} \vee z$, while $x \wedge y \vee \overline{z} \wedge x \vee \overline{y} \vee z$ is not.