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Problem set 9
June 15, 2015
Summer Semester 2015

Online and Approximation Algorithms

Due June 22, 2015 before class!

Exercise 1 (EXPO - 10 points)

Recall the online search problem presented in class and the EXPO algorithm for solving it. Moreover, let μ be a probability distribution of the natural numbers $\mathbb{N}$ according to which the number i is chosen with probability q_i. Now, consider the EXPO(μ) algorithm which is defined as follows: Choose the price $p \cdot 2^i$ with probability q_i, where p is the first price revealed.
Show that EXPO(μ) is $\frac{2}{q_j}$-competitive, for some $j \in \mathbb{N}$.

Exercise 2 (Experts - 10 points)

We are advised by N experts and we want to make predictions on whether the stock market goes up or down for a period of T days. At the beginning of each day $t \in [1, T]$, we get an advice from each expert which is either up or down. Based on this advice, we make our prediction and, at the end of the day, we learn if our prediction was correct. Our objective is to minimize the number of mistakes. Propose an online strategy for making at most $(M + 1) \log N$ mistakes, where M is the total number of mistakes made by the best expert during $[1, T]$.

Exercise 3 (k-Server - 10 points)

Show that the following greedy algorithms are not competitive for the k-server problem.

- Serve each incoming request by moving the server which is closest to it.
- For each server s_j, remember the distance D_j traveled by it. Serve each incoming request σ_i by moving the server s_j with the minimum $D_j + d(p_i, \ell(s_j))$, where p_i is the point of request σ_i and $\ell(s_j)$ is the location of s_j.

Exercise 4 (k-Server on a Line - 10 points)

Consider the k-server problem where all servers and requests are located on a continuous straight line. Algorithm DC (*Double Coverage*) serves each incoming request on the point x as follows:

- If x is on the left of all servers, move the closest server to it. Treat similarly the case where x is on the right of all servers.

- Otherwise, x is located between two servers s_i and s_j. Move both servers with equal speed towards x until one of them reaches x (i.e. if s_i is the closest, then they both move distance $d(s_i, x)$).

The aim of the exercise is to show that Algorithm DC is k-competitive for the k-server problem on a line. Let $s_1, s_2, \ldots, s_k$ and $a_1, a_2, \ldots, a_k$ be the locations of the servers by DC and OPT, respectively. We define the potential function $\Phi = k \cdot M + D$, where M is the minimum cost perfect matching in the bipartite graph between $s_1, s_2, \ldots, s_k$ and $a_1, a_2, \ldots, a_k$, while $D = \sum_{i<j} d(s_i, s_j)$ is the sum of all pairwise distances between the servers of DC.

- Show that Φ satisfies the following properties:
  - If the adversary's cost increases by y, then the change in the potential is $\Delta\Phi \leq k \cdot y$.
  - If the cost of DC increases by y', then the change in the potential is $\Delta\Phi \leq -y'$.
- Show that DC is k-competitive.