Technische Universität München
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Wintersemester 2015-16
Assignment 5
November 16, 2015

Efficient Algorithms and Datastructures I

Question 1 (10 Points)

Solve the following recurrence relation using a generating function:
$a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ with $a_0 = 0$ and $a_1 = 1$.

Question 2 (10 Points)

(a) Describe how to implement a queue using two stacks and $O(1)$ additional memory, so that the amortized time for any ENQUEUE or DEQUEUE operation is $O(1)$. The only access you have to the stacks is through the standard subroutines PUSH and POP.

(b) A quack is a data structure combining properties of both stacks and queues. It can be viewed as a list of elements written left to right such that 3 operations are possible:

(i) QPUSH: add a new item to the left end of the list

(ii) QPOP: remove the item on the left end of the list

(iii) QPULL: remove the item on the right end of the list

Implement a quack using 3 stacks and $O(1)$ additional memory, so that the amortized time for any QPUSH, QPOP, or QPULL operation is $O(1)$. Again, you are only allowed to access the stacks through the standard functions PUSH and POP.

Question 3 (10 Points)

Suppose instead of using decimal or dual representation of numbers, we represent them in binary over the basis of Fibonacci numbers. That is, the bit-string $(X_k, X_{k-1}, \cdots, X_1)_F$ represents the number $n = \sum_{i=1}^{k} X_i \cdot F_i$, where F_i denotes the ith Fibonacci number ($F_1 = F_2 = 1$ and $F_i = F_{i-1} + F_{i-2}$ for $i \geq 3$). For example, $(31)_{10}$ can be represented by the bit string $(10100100)_F$ since $F_8 + F_6 + F_3 = (21)_{10} + (8)_{10} + (2)_{10} = (31)_{10}$, and also by the bit string $(10011011)_F$ since $F_8 + F_5 + F_4 + F_2 + F_1 = (21)_{10} + (5)_{10} + (3)_{10} + (1)_{10} + (1)_{10} = (31)_{10}$.

(a) Argue that we can represent any number $n \in \mathbb{N}_0$ like this.

(b) Describe an algorithm which performs increment and decrement operations in this representation in constant amortized time (starting from 0). Assume that flipping each bit requires one unit of work.
(*Hint*: Use a potential function that assigns potential depending on whether consecutive pairs of bits are similar. For example, if bit i and bit $i + 1$ are equal/different, you may say they contribute one unit to the potential. Make sure that the potential of $(0)_F$ is zero units.)

Question 4 (10 Points)

Consider a sequence containing the characters a,b,c,d,e,f,g with probabilities (number of times a character appears/ total length of sequence) 0.01, 0.24, 0.05, 0.2, 0.47, 0.01, 0.02 respectively. Construct a static binary tree which minimizes the cost for accessing elements in this sequence.

Extra:Now construct the complete BST containing these characters such that the inorder traversal corresponds to the alphabetical order. On this splay tree, access the following characters: g,c,e