
Efficient Algorithms and Datastructures I

Question 1 (10 Points)

n motorcyclists M_1, M_2, \dots, M_n start riding their bikes from a (straight) start line. At the start M_i and M_{i+1} are adjacent to each other. Each motorcyclist M_i starts at some angle ϕ_i and keeps riding in a straight line along this direction at a constant speed $s_i > 0$. Whenever a motorcyclist M_j comes across the path traversed by any other motorcyclist M_i , we say that M_i defeated M_j and in that case, M_j stops riding.

- (a) We call the point where M_i defeats M_j as the point of ambush $A_{i,j} \in \mathbb{R}^2$. Show that if $A_{i',j'}$ is a point of ambush which occurs closest to the start line, then i' and j' are consecutive integers.
- (b) Show how to enumerate in $O(n \log n)$ time, all events where one motorcyclist defeats another.

Question 2 (10 Points)

For any positive integer n , show a sequence of Fibonacci heap operations that creates a Fibonacci heap consisting of just one tree that is a linear chain of n nodes.

Question 3 (10 Points)

Give a sequence of m MAKESET, UNION and FIND operations, n of which are MAKESET operations, that take $\Omega(m \log n)$ time when we use union by rank only.