Technische Universität München
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Theoretical Computer Science

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Worksheet
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Fundamental Algorithms 7

Exercise 1

Let $n = 1000$. Compute the values of the hash function $h(k) = \lfloor n(ak - \lfloor ak \rfloor) \rfloor$ for the keys $k \in \{61, 62, 63, 64, 65\}$, using $a = \frac{\sqrt{5}-1}{2}$. What do you observe?

Exercise 2

Given is a hash table `T[0,...8]` of 10 elements. Draw an image of this hash table after the keys 5, 28, 19, 15, 20, 33, 12, 17, and 10 have been inserted (in that particular order). Use the hash function $h\colon U \to \{0, 1, \ldots, 8\}$, $h(k) = k \bmod 9$, and use chaining to resolve collisions.

Exercise 2a

Repeat exercise 2 for hash tables that use open addressing. Use a hash table `T[0,...10]` with 11 elements, instead, and use the following hash functions:

(1) $h(k, i) := (k + i) \bmod 11$

(2) $h(k, i) := \left(k \bmod 11 + 2i + i^2\right) \bmod 11$

(3) $h(k, i) := (k \bmod 11 + i\,(k \bmod 7 + 1)) \bmod 11$

Insert the keys 5, 19, 27, 15, 30, 34, 26, 12, and 21 (in that order). State which keys require the longest probe sequence in the resulting tables.

Exercise 3

Consider a universe U of keys, where $|U| > mn$, and a hash function $h\colon U \to \{0, 1, \ldots, n-1\}$. Show that there is at least one subset of U that contains m keys that are all hashed to the same slot by h.