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Abstract

We present a new and simple algorithm to reconstruct suffix links in suffix trees and suffix arrays. The algorithm is based on observations regarding suffix tree construction algorithms. With our algorithm we bring suffix arrays even closer to the ease of use and implementation of suffix trees.

Keywords: Pattern Matching, Suffix Links, Suffix Trees, Suffix Arrays

1 Introduction

Historically, suffix links were an invention to facilitate linear-time¹ construction of suffix trees [Wei73, McC76, Ukk95]. It has since been discovered that suffix links have uses of their own (e.g., [Gus97]), most notable for the computation of matching statistics and approximate pattern matching [CL94]. Other applications are finding tandem repeats in linear time [GS04] or the construction of DAWGs [Gus97]. Due to the large size of the suffix tree, suffix links are often discarded or not even constructed. Giegerich and Kurtz [GKS03] have proposed a very space efficient method for top-down construction of suffix trees that does not use suffix links. In Farach's construction method for large alphabets suffix links are not used either [Far97]. Furthermore, recent developments have made the suffix array [MM93] a much more interesting data structure. Kasai et al. [KLA⁺01] have shown how the suffix array can be used to simulate a bottom-up suffix tree traversal and how to compute the longest common prefix information in linear time (see also [Man04]). Abouelhoda et al. [AKO02, AOK02, AKO04] have enhanced the suffix array so that it can be used with the same asymptotically optimal time bounds as suffix trees in exact matching and other applications. In [AKO04] two methods for suffix link reconstruction are proposed, one with linear-time complexity using (complex) lowest common ancestor (LCA) data structures (see, e.g., [BFC00, BFCP⁺01, Sad02]), and another simpler one that has complexity $O(n \log n)$. Kim et al. [KJP04] use suffix links on the enhanced suffix array to merge two suffix arrays in linear time for integer alphabets. They also give a linear-time algorithm that reconstructs suffix links. The algorithm does not need constant time LCA data structures. On the other hand, it uses $2n$ lists and bucket sorting, and it is therefore less space efficient than our algorithm, which uses only one additional integer array.

¹We assume a uniform cost model throughout this paper.

Linear-time algorithms for suffix array construction have been introduced by Kärkkäinen and Sanders [KS03], Kim et al. [KSPP03], Ko and Aluru [KA03]. For practical use, Larsson and Sadakane [LS99], Manzini and Ferragina [MF04], and Burkhardt and Kärkkäinen [BK03] have presented some very fast – but asymptotically non-linear – algorithms which seem to outperform the linear ones (see [ARS⁺04, PST05] but also [LP04]). Grossi and Vitter have introduced the compressed suffix array [GV00] and another succinct representation, the FM-index, has been developed by Ferragina and Manzini [FM00]. Based on the compressed suffix arrays Sadakane describes a succinct implementation of suffix trees that uses linear space (in bit complexity) and also offers suffix links [Sad04]. The drawback of the compressed suffix arrays is that their practical performance is much worse than that of normal suffix arrays. From the empirical studies in [SS01] and [Kai04] one can expect an increase in the running time by a factor of twenty when comparing normal to compressed suffix arrays.

Our contribution is a very simple, easy-to-implement, and efficient algorithm to reconstruct suffix links on suffix trees and (enhanced) suffix arrays. Under the uniform cost model the algorithm has linear time and space complexity. It is much simpler than the algorithms based on LCA computation because it only does two simple depth first search (DFS) traversals of the tree structure. It is alphabet independent and can thus be used with integer alphabets, for instance, together with Farach-Colton's suffix tree construction algorithm. Furthermore, it can be seen as a simple enhancement of the enhanced suffix array, making the algorithm [AKO04] run in linear time without the need for LCA or range minimum query (RMQ) data structures.

2 Algorithm

We assume that the reader has basic knowledge in suffix trees, i.e., for an easy understanding, the reader should know the suffix tree construction algorithm by Ukkonen [Ukk95].

In the following, let Σ be an arbitrary alphabet. Note that we do not require a finite alphabet. Let Σ^* denote the set of all finite strings over Σ (including the empty string ε). Let $t = t_1 \cdot \dots \cdot t_n \in \Sigma^n$ be a string of length $|t| = n$. If $t = uvw$ with $u, v, w \in \Sigma^*$ then u is a prefix, v a substring, and w a suffix of t . We define the i -th suffix $\text{suff}_i(t) = t_i \cdot \dots \cdot t_n$. Following Giegerich and Kurtz [GK97] we define a Σ^+ -tree T as a rooted, directed tree with edge labels from Σ^+ . For each $x \in \Sigma$, every node in T has at most one outgoing edge whose label starts with x . A Σ^+ -tree T is called compact, if all nodes are either the root, leaves or branching. For a node p let $u \in \Sigma^*$ be the string that is constructed by concatenating all edge labels on the path from the root to the node p . We define the path of p as $\text{path}(p) = u$ and the string depth of p as $\text{depth}(p) = |u|$. For a Σ^+ -tree T , let its word set $\text{words}(T)$ be all strings u for which there exists a node p with $\text{path}(p) = uv$ for some $v \in \Sigma^*$. The suffix tree $\text{CST}(t)$ of a string t is defined as the compact Σ^+ -tree T with $\text{words}(T) = \{u \mid u \text{ is a substring of } t\}$. For each leaf p of T we define the leaf index $\text{lindex}(p)$ of p to be i if $\text{path}(p) = \text{suff}_i(t)$, i.e., p represents the i -th suffix of t . For node p let $\text{leaves}(p)$ be the leaves in the subtree rooted at p . We let \perp stand for an undefined value (as an undefined value for a pointer to a node).

A suffix link is an auxiliary edge of a Σ^+ -tree T . A suffix link points from a node p to a node q which by $\text{path}(q)$ represents the shortest (proper) suffix of $\text{path}(p)$ in T . In suffix trees, usually only suffix links for inner nodes are added. Here, we have the property that $\text{path}(p) = x\text{path}(q)$ for some character $x \in \Sigma^+$.

In the following let T be a suffix tree $\text{CST}(t)$ for string t of length n . Our algorithm constructs

suffix links solely based in the structure of the suffix tree and the leaf indices, therefore the size of the alphabet does not matter. The intuitive idea is to construct an array A of size n that contains the branching nodes in the same order as they are encountered by a suffix tree construction algorithm of McCreight [McC76] or Ukkonen [Ukk95]. We can then almost reconstruct the suffix links by setting for each node in A the suffix link to the successor in A . This takes care of almost all cases but those where the “active prefix” [Ukk95] has grown. Thus, we modify the idea to find for each node the index of the leaf that has “caused” the node. From there we find the corresponding branch for the next leaf because we know its leaf index and the branch depth.

For a node q of T let its minimal index be the minimal value of a leaf index of a leaf in the subtree rooted at q . For an inner node p of T let S be the set of minimal indices of its children. We define $\text{cause}(p)$ as the second smallest element in S . Formally,

$$\text{cause}(p) =_{def} \min_2 \left\{ i \mid i = \min_{r \in \text{leaves}(q)} \text{lindex}(r) \right\},$$

where $\min_2 I$ yields the second smallest element in the set I . Further, for a leaf p we define $\text{branch}(p, d)$ as the ancestor of p at string depth d , i.e.,

$$\text{branch}(p, d) =_{def} \begin{cases} q & , \text{ if } q \text{ is an ancestor of } p \text{ and } \text{depth}(q) = d \\ \perp & , \text{ otherwise.} \end{cases}$$

The correctness of our algorithm follows from the next lemma. Note that for every node p of a suffix tree with $\text{path}(p) = xu$ there exists a node q with $\text{path}(q) = u$ to where the suffix link of p will be pointed (see, e.g., Giegerich and Kurtz [GK97]).

Lemma 1 (Suffix Links). *Let p be a non-leaf node of the suffix tree T for a string $t \in \Sigma^n$. The suffix link of p is $\text{branch}(q, \text{depth}(p) - 1)$, where q is the leaf with $\text{lindex}(q) = \text{cause}(p) + 1$.*

Proof. Let $xu = \text{path}(p)$, $u \in \Sigma^*$, $x \in \Sigma$. The suffix link of p must point to a node r with $\text{path}(r) = u$. By definition, $\text{cause}(p)$ is a leaf index from a leaf in the subtree of p . Thus, xu is a prefix of $\text{suff}_{\text{cause}(p)}(t)$ and u is a prefix of $\text{suff}_{\text{cause}(p)+1}(t)$. The length of u is $\text{depth}(p) - 1$. Hence, u is a prefix of $\text{suff}_{\text{lindex}(q)}(t)$ of length $\text{depth}(p) - 1$ and u is represented by an ancestor $r = \text{branch}(q, \text{depth}(p) - 1)$ of q with $\text{depth}(r) = \text{depth}(p) - 1$. \square

Pseudo code for the algorithm is given in Figure 1. The complexity of the algorithm is given as follows.

Theorem 1 (Correctness and Complexity). *Algorithm $\text{main}(T)$ correctly computes the suffix links for the suffix tree T in time and space $O(n)$.*

Proof. The correctness follows directly from Lemma 1 and the fact that for any two nodes p and q $\text{cause}(p) \neq \text{cause}(q)$. The latter is easy to see as we take the second smallest value from the smallest values from children below. This value is never passed on to a parent.

The complexity follows directly from the algorithm. We perform two DFSs and take constant time per node. The space used is that for two arrays of size at most n and the (call) stack (also of size at most n). \square

<pre> prepare(p, A) if p is a leaf then Let i be the leaf index of p return(i) else Let d be the depth of p $min_1 := n + 1$ $min_2 := n + 1$ for all children q of p do $m = \text{prepare}(q, A)$ if $m < min_1$ then $min_2 := min_1$ $min_1 := m$ else if $m < min_2$ then $min_2 := m$ end if end for Set <math>A[$min_2 + 1$] := p</math> return(min_1) end if </pre>	<pre> compute(p, A, B) if p is a leaf then Let i be the index of p if $A[i] \neq \perp$ and $A[i]$ is not the root then Let d be the depth of $A[i]$ Set suffix link of $A[i]$ to $B[d - 1]$ end if else Let d be the depth of p Set $B[d] := p$ for all children q of p do compute(q, A, B) end for end if </pre> <hr style="border: 0.5px solid black;"/> <pre> main(T) Let r be the root and h the height of T Let A be an array of size n initialized to \perp Let B be an array of size h initialized to \perp prepare(r, A) //bottom-up traversal compute(r, A, B) //top-down traversal </pre>
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Figure 1: Pseudo code for the suffix link reconstruction algorithm. Array A corresponds to cause through the equation $A[\text{cause}(p) + 1] = p$ and array B corresponds to branch through the equation $B[d] = \text{branch}(p, d)$. In $\text{prepare}(p, A)$ we take the minimal leaf indices for each subtree and compute the minimum (which we pass back) and the second minimum (which we store in A). In $\text{compute}(p, A, B)$ we compute branch in B on the fly and use the cause values in A to set the suffix links.

3 Implementation Issues

For suffix trees the implementation is straight forward. For the case that the depth of a node is not stored explicitly, we need an additional array AH to accompany A for storing the string depth of the nodes in A . The height of the tree – if not readily available – can be computed during the first DFS with only small modifications. It is not needed for the asymptotic result because we can simply use n as an upper bound when creating the array B . Since the height of a suffix tree is $O(\log n)$ on average, less memory will be used in practice. Note also, that it might be possible to store the depth in array AH in a smaller field (i.e., a byte).

For suffix arrays we can use the method of Kasai et al. [KLA⁺01] for the bottom-up traversal ($\text{prepare}(r, A)$). For the top-down traversal ($\text{compute}(r, A, B)$) we need the additional data structures introduced by Abouelhoda et al. [AKO02]. Normally, a suffix tree node is identified by two borders. It is more practical to have a single identifier. This can be generated in an additional array ld through a top-down traversal. For each interval $[l, r]$ we store either l in $ld[r]$ or r in $ld[l]$. Using a single bit this can be marked accordingly and we can use the index in ld as an identifier for $[l, r]$. Observe that in a top-down traversal for each node (interval) either l or r is not yet used: The child intervals are always smaller than the parent intervals. If no index were free, we would have used the left and right border for two ancestor intervals $[l, r']$ and $[l', r]$. W.l.o.g., let $[l, r']$ be a child of $[l', r]$, then $r \geq r' \geq r$, i.e.,

$r = r'$. This is a contradiction to $[l, r']$ being an ancestor of $[l, r]$.

Thus, we use an array *ld* for node (interval) identifiers and an array *sl* for suffix links. Using the enhanced suffix array data structures [AKO04] it is also possible to retrieve the depth of a node in constant time. The suffix link creation can thus be implemented to use only one additional temporary array during construction (plus some stack size). If all arrays are implemented naively, the total structure takes five integers (*sa*, *lcp*, *childtab*, *ld*, *sl*) plus one integer (*A*) per text character and a logarithmic amount of space during construction (the average size of *B* and the stack). The structure is equivalent to a suffix tree, whose most efficient implementation also takes twenty bytes in the worst case [Kur99]. A more elaborate version would take advantage of the fact that most values in the arrays are small or can be made small by storing relative distances. Abouelhoda et al. [AKO04] report that in this way it is possible to reduce the size of *lcp* and *childtab* to one byte per character. The same is possible for *ld*. As a result we get an implementation using eleven bytes per text character plus some small (i.e., logarithmic in *n*) amount of additional space. The latter is comparable even to the worst-case size of the suffix tree data structure described in [GKS03].

We conducted some simple testing to compare the above described enhanced suffix array (ESA) with the currently most memory efficient linear-time suffix tree data structure (ILLI) by Kurtz [Kur99]. We used Ukkonen's algorithm [Ukk95] for suffix tree construction and Manzini and Ferragina's algorithm [MF04] for suffix array construction. The latter is not asymptotically linear but has a very good performance in practice [Maa05] (better than asymptotically linear algorithms).

For our tests we used a set genetic sequences available via GenBank (which have the identifiers NC_001460, NC_001454, NC_004001.2, NC_002067, NC_001405, NC_003266, U47924, AC002397, L43967, NC_000912, BA000008, AE002161, NC_000922, NC_003098.1) and the Calgary Corpus². The cumulated results measured on an AMD Athlon XP1800+ with 1544.732 MHz and 1 GB of main memory are shown in Figure 2. With all its new gadgets the suffix array has become fat, fatter even than the suffix tree when we take a look at the maximal memory used during construction. On the other hand, the suffix array has become faster and, as a result, comparable to the suffix tree in our implementation. Only when including the highly repetitive file "pic" from the Calgary Corpus the coding scheme leads to large time and size penalties. We further witness the effect that a larger alphabet size results in a smaller size of the suffix tree and a larger construction time (due to the linked list implementation). As a result, we find that the difference between using a suffix tree or a suffix array has become very small. The rule of thumb that suffix arrays are smaller but slower does not count any more if one wants "full functionality".

4 Conclusion

The last years have seen a major change in the focus of the suffix data structures. The suffix array is becoming more and more the most adequate data structure being preferred to the suffix tree. This is mostly due to the demand for indexing huge texts. Suffix trees and arrays have converged towards each other as a result of the space reduction of suffix trees and the running time enhancement of suffix arrays. We hope to have added another small part to this convergence process. As the difference between both data structures has become very small, it would be interesting whether one could describe their relation and derive fundamental differences that come from the two different concepts of using intervals and nodes.

²Available from <ftp://ftp.cpsc.ucalgary.ca/pub/projects/text.compression.corpus>.

Algorithm	Input	Average time			Average ratio	
		construction	search	traversal	memory-usage/input-size maximum	index-only
ESA	GB	1.57 s	1.25 s	0.77 s	16.13	12.10
ILLI	GB	1.37 s	0.95 s	0.78 s	13.07	13.07
ESA	CC	11.49 s	1.38 s	0.56 s	16.72	12.48
ILLI	CC	0.48 s	0.39 s	0.24 s	9.97	9.97
ESA	CC\pic	0.35 s	0.29 s	0.22 s	16.41	12.27
ILLI	CC\pic	0.48 s	0.33 s	0.22 s	9.99	9.99

Figure 2: Comparison of the enhanced suffix array and the space efficient suffix tree with genomic data (GB) and the Calgary Corpus (CC). The table gives average values. After the index was constructed we performed 1000 random searches with words from the index text to test the search performance and 100 matching statistic like traversals with the text itself to test the suffix link performance. The maximum memory usage per input character is met during construction. After construction some memory could be freed for the suffix array. Thus, the last column gives the operating size per input character. Note that the text itself contributes another byte to the size. The Calgary Corpus contains a file “pic” with a very high redundancy which lead to a lot of nodes with a depth greater than 255 (the size fitting one byte). The simple size reduction scheme for the lcp array does not work here as more than half of the values stored were larger than 255. This resulted in an extra table with four times the size of the small lcp array.

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